# Courant Research Centre

## ‘Poverty, Equity and Growth in Developing and Transition Countries: Statistical Methods and Empirical Analysis’

**Georg-August-Universität Göttingen**
**(founded in 1737)**

Discussion Papers

**No. 108**

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**R. Emre Aytimur, Aristotelis Boukouras, Robert Schwager**

**January 2012**

Wilhelm-Weber-Str. 2 · 37073 Goettingen · Germany
Phone: +49-(0)551-3914066 · Fax: +49-(0)551-3914059

Email: crc-peg@uni-goettingen.de Web: http://www.uni-goettingen.de/crc-peg

# Voting as a Signaling Device

R. Emre Aytimur*, Aristotelis Boukouras†, Robert Schwager‡

November 22, 2011

**Abstract**

In this paper, citizens vote in order to influence the election outcome and in order to signal their unobserved characteristics to others. The model is one of rational voting and generates the following predictions: (i) The paradox of not voting does not arise, because the benefit of voting does not vanish with population size. (ii) Turnout in elections is positively related to the size of the local community and the importance of social interactions. (iii) Voting may exhibit *bandwagon effects* and small changes in the electoral incentives may generate large changes in turnout due to signaling effects. (iv) Signaling incentives increase the sensitivity of turnout to voting incentives in communities with low opportunity cost of social interaction, while the opposite is true for communities with high cost of social interaction. Therefore, the model predicts less volatile turnout for the latter type of communities.

**Keywords:** electoral incentives, signaling, voting

**JEL Classification:** C70, D72, D80

*Chair of Public Economics, Georg-August University Göttingen, raytimu@gwdg.de

†Courant Research Centre Poverty, Equity and Growth, Georg-August University Göttingen, aboukou@gwdg.de

‡Chair of Public Economics, Georg-August University Göttingen, rschwag@uni-goettingen.de

# 1 Introduction

What motivates citizens to vote is one of the fundamental questions of political science and public economics. Since the early writings of Downs (1957) and later on Ledyard (1984), the rational-choice theory puts the desire of citizens to affect the election outcome as the main driving factor of their voting behavior. But, since the probability to actually change the outcome is very small, the instrumental view of voting generates the **paradox of not voting**[1], which has led many researchers to propose different reasons that drive voting incentives.

The purpose of this paper is to provide a formal model of voting as a signaling device, and, in doing so, to provide a rational choice model which does not generate the paradox of not voting. The main idea is that citizens possess unobserved characteristics, such as their preferences for public goods or their degree of altruism, which they signal to others through voting. If informative, these signals benefit both the sender and the receiver, because they facilitate the creation of mutually beneficial cooperations or because they increase the trust in an already given relation. Examples of cooperations are exchanging information about job opportunities, helping each other to take care of daily issues, etc.

More specifically, we consider a two-period extension of Borgers (2004) model in a finite-agent economy, which is divided into neighborhoods. In the first period citizens decide to vote or not and they also observe whether their neighbors voted. In the second period, after mutual agreement, each citizen can form partnerships with any of her neighbors[2]. Citizens derive utility from both the outcome of the election, as in the instrumental view, and the formation of partnerships in the second stage. Their utilities, however, are a function of two unobservable characteristics: (i) their cost of voting and (ii) a preference parameter, the latter affecting the utility from both the election outcome and the partnership. The parameter can be interpreted as either representing the intensity of preferences for public goods or as representing the degree of one's altruism. The crucial assumption is that the utility of the election outcome is correlated with the utility of the partnership (the assumption that this correlation is perfect in our model is not so important and can be relaxed). Because it is costly to form partnerships, a citizen is willing to cooperate with her neighbor only if the

[1]For a formal treatment, see Palfrey and Rosenthal (1985).

[2]An alternative interpretation of the second stage is that each citizen has already a network of friends and each one of them decides whether to increase the degree of interaction with her friends or not.

latter has a high intensity of preferences for public goods. As a result, citizens' voting incentives are enhanced by their willingness to signal their preferences for cooperation to their neighbors.

We find the perfect Bayesian equilibria of the game and we analyze the most interesting case: stable interior equilibria with signaling, that is stable equilibria where a fraction of agents from every type votes. We show that such equilibria exist and we compute their comparative statics. The main results are as follows:

1. The presence of signaling strictly increases voting incentives and electoral turnout when compared to models without signaling effects, like Borgers (2004). This is a direct implication of the value of signaling and the utility that citizens receive from social interactions.

2. Even in economies with very large populations, the value of signaling does not tend to zero and therefore the paradox of not voting does not arise (or more precisely the set of parameter values, according to which non-voting equilibria exist, shrinks).

3. Communities with closer personal ties and higher level of social interaction present higher turnout.

4. Due to signaling, electoral incentives may exhibit "bandwagon" effects: the benefit of voting may increase with turnout, so that one's willingness to vote increases if the expected participation rate increases. To the best of our knowledge, this is in contrast to existing papers on rational voting, where the benefit of voting is always decreasing with turnout due to the decreasing pivotal probability.

5. Signaling incentives interact with direct electoral incentives so that even a small change in the importance of the election may generate a sizable increase in turnout. This is because turnout may be highly sensitive to signaling effects. In particular, in countries with low cost of social interaction (low opportunity costs of time, bad substitutes to social interaction), the presence of signaling increases the sensitivity of turnout to electoral incentives. On the other hand, in countries with high cost of social interaction, the presence of signaling decreases the sensitivity of turnout to electoral incentives. In terms of empirical predictions, the model suggests that communities with high cost of social interaction should have *lower* volatility of turnout in response to changes in the importance

of elections than communities with lower costs of social interactions. In terms of policy, the model predicts that increasing the value of the election (through increasing the awareness of citizens about the policy agenda or through political advertising) has a higher impact on electoral turnout in communities with lower interaction costs and closer community ties.

The model captures in a simple way the interaction between electoral and social incentives, which we believe is an important driving force of voting incentives. A growing number of empirical papers (Gerber, Green, and Larimer (2008), Funk (2010), see also below) show that social considerations and pressures play an important role in citizens' voting decisions. Also our study is motivated by common experience and intuition. Neighbors often cooperate to provide local public goods, like taking kids to school or taking care of communal spaces, while friends and colleagues engage in mutually beneficial interactions, like information sharing and undertaking of small favors. Signaling one's good will and trustworthiness can thus have significant value when compared with relatively low cost activities, like voting. Our model formalizes this intuition and shows that the effects on direct electoral incentives can actually be large.

There are many strands of the literature which are related to our paper. Overbye (1995), Posner (1998) and Bufacchi (2001) also argue that reputation and signaling reasons can account for the voting behavior of citizens in modern democracies, but they provide no formal analysis. By constructing a rigorous model formalizing this idea, we are able to make testable predictions which relate the voting behavior to the social conditions of individuals.

Also, Funk (2005) analyzes a voting model with signaling incentives. However, there are two main differences between her paper and ours. First, in her model voting takes place in order to signal one's willingness to comply with social norms, while in our model the signaling concerns one's ability to cooperate in mutually beneficial interactions. Second, the main focus of our analysis is the interaction between electoral and signaling incentives, while Funk (2005) ignores electoral incentives and focuses on the impact of new technologies, which reduce the cost of voting, to signaling incentives and turnout.

Other papers, such as Edlin, Gelman, and Kaplan (2007), Fowler (2006) and Rotemberg (2009), argue that social preferences and altruism are the main driving forces of voting behavior. While our model does not focus on this explanation, one of the interpretations of the citizens' unobserved parameter is that it represents social preferences.

However, this parameter generates two voting effects in our model: one direct and one indirect, through signaling. The second channel, which is our main focus of study, is absent from the social preferences literature.

There exist several other theoretical approaches to voting incentives. According to the *ethical voting* literature (Harsanyi (1980), Coate and Conlin (2004), Feddersen and Sandroni (2006)) voters decide on the ground of moral principles and they derive utility from adhering to them. The *leader-follower* theories (Uhlaner (1989), Morton (1991), Shachar and Nalebuff (1999), Herrera and Martinelli (2006)) emphasize the role of leaders and their ability to impose sanctions or to provide rewards in motivating social groups to participate in elections. Castanheira (2003) argue that voting benefit can be high, since the implemented platform after the elections depends not only on the winner, but also on the margin of victory. Papers on *expressive voting* (Brennan and Hamlin (1998), Engelen (2006)) argue that voting is a consumption good in itself, because it allows one to affirm her own beliefs and values. Contributions to the literature on *social norms* (e.g., Coleman (1990)) point out that voting is a public good in itself and show how social norms are used to overcome the associated free-rider problem.[3]

We do not question the relevance of these approaches. Rather, the theory presented here provides an additional rationale for voting, which may complement the arguments put forward in existing literature, and which has not been analyzed so far.

Finally, our model generates predictions which are consistent with empirical and experimental results. An increasing number of papers finds that social pressure, close community ties and voter participation increase the voting incentives for community members. Gerber, Green, and Larimer (2008) show through a large-scale field experiment that turnout was substantially higher among people who received a letter before the elections, which was explaining that whether they voted or not would be made public among the neighbors. Funk (2010) finds that voter turnout was negatively affected in small communities of Switzerland after the introduction of postal voting. Her explanation is that although postal voting decreased the voting costs, it also removed signaling benefit of voting, which was substantial in small communities. Gerber and Rogers (2009) find that a message publicizing high expected turnout is more effective at motivating people to vote than a message publicizing low expected turnout. This result in spite of lower pivotal probability with higher turnout is consistent with the signaling benefit and the bandwagon effect of our model. Similarly, an experiment of

[3]For more complete surveys, see Aldrich (1993), Blais (2000), Dhillon and Peralta (2002), Feddersen (2004).

sequential voting by Großer and Schram (2006) shows that high turnout of early voters increases late voters' turnout.[4]

The paper proceeds as follows. Section two presents the model, section three provides the equilibrium analysis, section four presents the main comparative statics and results and section five includes the final comments and conclusions. Most proofs are relegated to the appendix.

# 2 The Model

There are $N$ individuals, $i = 1, 2, ..., N$, and two political parties, $A$ and $B$. Each individual is summarized by three characteristics. The first one is the preferred party of the individual $i$: $R_i \in \{A, B\}$. The second one is his cost of voting, $c_i \in [c_{min}, c_{max}]$ with $0 \leq c_{min} < c_{max}$. The last characteristic is whether he is of high or low type, $\tau(i) \in \{H, L\}$, which refers to the importance the individual $i$ attaches to decisions taken in the public domain. Each characteristic of any individual $i$ is a random variable. All three characteristics are independently distributed for each individual and across individuals. The preferred party of any individual $i$ is $A$ with probability $1/2$ and $B$ with probability $1/2$. The cost of voting $c_i$ of any individual $i$ is distributed according to the cdf $F$ on the support $[c_{min}, c_{max}]$ with the pdf $f$ which is positive on all of the support. Finally, any individual is of high type, $\tau(i) = H$, with probability $q$ and of low type, $\tau(i) = L$, with probability $1 - q$. Each individual privately knows his characteristics. The distributions of individuals' characteristics are common knowledge.

There are two periods. In the first period, the election occurs in which an individual chooses to vote for his preferred party or to abstain.[5] The winner is determined by a simple majority rule. In case of a tie, each party wins with probability $1/2$.

An individual $i$'s payoff from the first period is as follows: His benefit is $w_1\alpha_{\tau(i)}$ if his preferred party wins and 0 otherwise. $w_1$ is a parameter which measures the importance of the election, such as the value of the public decision to be determined by the winner of the election. We assume that both types care about the result of the election, as measured by the parameter $\alpha_{\tau(i)}$, and that a high type individual cares more about it than a low type individual, i.e. $\alpha_H > \alpha_L > 0$. His cost is $c_i$ if he votes and 0 otherwise. Hence, if he votes and his preferred party wins, his payoff is $w_1\alpha_{\tau(i)} - c_i$. If he abstains

[4] For other papers which study the relation between social interactions and political participation, see for instance Schlozman, Verba, and Brady (1995) and Schram and Sonnemans (1996).

[5] Since voting for the other party is a weakly dominated strategy, we do not consider this strategy.

and his preferred party wins, his payoff is $w_1\alpha_{\tau(i)}$. If he votes and his preferred party loses, his payoff is $-c_i$. If he abstains and his preferred party loses, his payoff is 0.

In the second period, social interactions occur in neighborhoods composed of $n$ individuals in the form of pairwise matches. After observing whether each one of his neighbors voted or not, an individual $i$ chooses to match or not with each individual $j = 1, 2, ..., n$, $j \neq i$. If both $i$ and $j$ agree to match with each other, they match together. Otherwise, a match does not occur.

An individual $i$'s payoff from a match with an individual $j$ depends both on his own type and his neighbor's type and we adopt the following simple interaction payoff:

$$w_2\alpha_{\tau(i)}(\alpha_{\tau(j)} - d) \tag{1}$$

where $d$ is the matching cost and $w_2$ measures the importance of social interactions. Equation (1) provides $i$'s payoff from a match with $j$, when $j$'s type is known to $i$. However, since $j$'s type is private information, $i$ needs to evaluate his expected payoff, after he has updated his belief about $j$'s type, given $j$'s voting choice. The formulation of the expected payoff of $i$ and the analysis of his best response are provided in the following section. We assume that $\alpha_L < d < \alpha_H$. Hence, in the perfect information case, an individual would agree (respectively, would not agree) to match with a high (respectively, low) type individual. Moreover, since $\alpha_H > \alpha_L$, if a match has a positive expected payoff, a high type individual has a higher expected payoff from this match than a low type individual. These "matches" or "interactions" are independent and non-exclusive, meaning that each agent can potentially interact with all of his neighbors if they also want to interact with him and the utility of each match is not affected by the other matches.

As in Borgers (2004), we make two symmetry assumptions about the voting strategy. We assume that it does not depend on the individual's preferred party and that all individuals play the same strategy of the form $s : \{H, L\} \times [c_{min}, c_{max}] \to \{0, 1\}$ where $s_i(\tau(i), c_i) = 0$ (respectively 1) means that an individual $i$ abstains (respectively votes) if he is of type $\tau(i)$ and his cost of voting is $c_i$.

Similarly to the voting strategy, we assume that the matching strategy does not depend on the individual's and on his potential partner's preferred parties and that every individual $i$ plays the same strategy of the form[6] $I : \{0, 1\} \to \{0, 1\}$ with regards

[6] The assumption that the matching strategy depends only on the potential partner's voting decision, and not on the individual's own type and own voting decision is not a restriction. The proof is available upon request.

to an individual $j$, $i \neq j$. $I(s_j) = 0$ (respectively 1) means that an individual $i$ does not agree (respectively agrees) to match with an individual $j$ if the individual $j$'s voting decision is $s_j$. Hence, a match between individuals $i$ and $j$ occurs if and only if $I(s_j)I(s_i) = 1$.

We interpret a high type individual as one who is more willing to contribute to the "public good" than a low type individual. That's why, agents want to match with a high type individual, and not with a low type individual. Interpreting the result of the election and social interaction processes as public goods, a high type individual gets a higher benefit in case of the victory of his preferred party and in case of a match. Therefore, his voting behavior can be a signal about his type, given that he is more likely to vote than a low type individual. The signal can be valuable since agents interact with each other if and only if they have posterior beliefs that the other one is of high type with a high enough probability.

Our equilibrium concept is subgame perfect Bayesian equilibrium. Hence, we proceed by backward induction.

# 3 Equilibrium Analysis

We first analyze the second stage of the game where agents decide whether to interact with each of their neighbors or not, after observing their voting behavior. Subsequently, we will use the equilibria of the second stage in order to analyze the first stage.

## 3.1 Second-Stage Equilibrium

Recall from the previous section that equation (1) provides $i$'s payoff from a match with $j$. The expected payoff of $i$ from a match with $j$, when $j$'s type is private information and conditional on $j$'s voting decision, is given by:

$$EP_{ij} = w_2 \alpha_{\tau(i)} \left[ \lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d) \right] I(s_i) I(s_j) \tag{2}$$

Here, $\lambda(s_j)$ is the posterior belief that a neighbor who voted ($s_j = 1$) or did not vote ($s_j = 0$) is of type $H$. For later use, we define $\lambda(1) = \lambda_H$, which is the posterior belief that a neighbor, who voted, is of high type, and $1 - \lambda(0) = \lambda_L$, which is the posterior belief that a neighbor, who did not vote, is of low type. $I(s_j)$, as given in the previous section, denotes the decision of agent $i$ (who has type $\tau(i)$) whether to match with neighbor $j$ or not, conditional on the latter's voting behavior ($s_j$). The overall second stage utility of $i$ from all his neighbors is simply the summation over all possible interactions in his neighborhood:

$$TEP_i = \sum_{j=1, j\neq i}^{n} \left\{ w_2 \alpha_{\tau(i)} \left[ \lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d) \right] I(s_i) I(s_j) \right\} \qquad (3)$$

The best-response of $i$ in the second stage of the game depends on the voting behavior of his neighbors and his posterior beliefs regarding their type. By (2), it is clear that the best-response for $i$ is to match with every neighbor who generates a positive interaction payoff and not to interact if the expected payoff is negative. Therefore, his best response is to interact if $\left[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d)\right] I(s_i) > 0$, not to interact if $\left[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d)\right] I(s_i) < 0$ and either if the expression is zero.

The above analysis suggests that there are multiple equilibria of the second stage, which depend on the sign of the expressions $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L - d$ and $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L - d$. For the remainder of the paper we focus on the most interesting of these equilibria, the one where agents choose to interact with only those neighbors who voted. This is the case when $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L \geq d$ and $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$.[7]

## 3.2 First-Stage Equilibrium

In this subsection, we compute the expected benefit of voting, given the second period equilibrium. Then, we show the existence of the most interesting equilibrium. We begin by the following remark:

In equilibrium, an individual votes if his expected benefit from voting exceeds his cost of voting. Since the benefit of voting is independent of the cost, an equilibrium voting strategy must be a threshold strategy like in Börgers (2004). So, there is some $c_H^*$ such that $s(H, c_i) = 1$ if $c_i < c_H^*$ and $s(H, c_i) = 0$ if $c_i > c_H^*$. Similarly, there is some $c_L^*$ such that $s(L, c_i) = 1$ if $c_i < c_L^*$ and $s(L, c_i) = 0$ if $c_i > c_L^*$. Hence, the ex ante probability that any individual votes is $p = qF(c_H^*) + (1 - q)F(c_L^*)$. For $0 < p < 1$, the posterior beliefs that a neighbor who voted is of high type, i.e. $\lambda_H$, and that a neighbor

[7]The analysis on all possible equilibria of the second game second is available by the authors upon request. We omit the analysis of all other cases either because they involve no signaling benefit (agents interact either with all agents or none) or because they are not possible when the first stage is considered (agents interact only with those who did not vote).

who did not vote is of low type, i.e. $\lambda_L$, are then given as follows by Bayes' rule:

$$\lambda_H = \frac{qF(c_H^*)}{qF(c_H^*) + (1-q)F(c_L^*)} \tag{4}$$

$$\lambda_L = \frac{(1-q)(1-F(c_L^*))}{(1-q)(1-F(c_L^*)) + q(1-F(c_H^*))} \tag{5}$$

The expected benefit of voting is the payoff difference between voting and abstaining from voting and it is the sum of two terms. The first one which we call the expected *electoral* benefit arises because one's vote can possibly change the electoral outcome. This is the standard benefit of voting in the literature. The second one which we call as the expected *signaling* benefit arises because one's vote can possibly change his outcome from the social interaction stage.

The expected electoral benefit of voting of an individual $i$ is positive only if individual $i$ is pivotal. This happens if his preferred party receives either the same number of votes as the other party or receives one less vote than the other party among the voters but him. In both cases, by voting for his preferred party, he increases the probability that his preferred party wins by 1/2. Taking into account that his benefit is $\alpha_{\tau(i)} w_1$ if his preferred party wins and 0 otherwise, we get that the electoral benefit of voting is equal to $\frac{1}{2}\alpha_{\tau(i)} w_1 \Pi(p)$, where $\Pi(p)$ is the probability that individual $i$ is pivotal. The expression of $\Pi(p)$ is as given in Borgers (2004), who also shows that this is a differentiable and decreasing function for all $p \in (0,1)$.

With respect to the expected signaling benefit, we should keep in mind that only voters match among each other. Hence, if an individual votes, he matches with all voters in his neighborhood. His expected payoff from a single match is $w_2\alpha_{\tau(i)}[\lambda_H\alpha_H + (1-\lambda_H)\alpha_L - d]$ and the expected number of voters (and so of matches) in his neighborhood but him is $p(n-1)$. Hence, if he votes, this gives him an expected payoff of $w_2\alpha_{\tau(i)}p(n-1)[\lambda_H\alpha_H + (1-\lambda_H)\alpha_L - d]$ in the second period. If he does not vote, he does not match with anyone, so his payoff is 0 in the second period. The payoff difference between the two cases where he votes or does not vote gives the expected signaling benefit of voting.

We denote by $B_{\tau(i)}(c_H, c_L)$ the total expected benefit of voting of an individual $i$ with type $\tau(i)$ as a function of the thresholds $c_H$ and $c_L$. An equilibrium is given by thresholds $c_H^*$ and $c_L^*$ such that $B_{\tau(i)}(c_H^*, c_L^*) \geq c_i$ for all $i$ who vote and $B_{\tau(i)}(c_H^*, c_L^*) \leq c_i$ for all $i$ who abstain.

For the second period equilibrium described in Section 3.1 (where only voters match among each other), we have the following total expected benefit of voting, where $p, \lambda_H, \lambda_L$ are functions of $c_H$ and $c_L$:

$$B_{\tau(i)}(c_H, c_L) = \alpha_{\tau(i)} \left\{ \frac{w_1}{2}\Pi(p) + w_2 p(n-1)[\lambda_H \alpha_H + (1-\lambda_H)\alpha_L - d] \right\} \tag{6}$$

We observe that $B_L = \mu B_H$ where $\mu = \alpha_L/\alpha_H < 1$. Depending on the levels of $c_{min}$ and $c_{max}$, there are many possible first-period equilibria. However, since $B_H > B_L$, turnout ratio of high type individuals is at least as high as turnout ratio of low type individuals in any equilibrium.[8]

We focus on the equilibria where a fraction of each type, strictly between zero and one, of individuals vote. We call these equilibria as *interior equilibria*. As already discussed in Section 3.1, our main interest being the signaling aspect of voting, we focus on interior equilibria with the second period equilibria where only voters match among each other. We call these equilibria as *interior equilibria with signaling*.

An interior equilibrium implies $c_L^* = \mu c_H^*$ since $B_L = \mu B_H$. Then, we can summarize the condition for an interior equilibrium as follows:

$$B_H(c_H^*, \mu c_H^*) = c_H^* \tag{7}$$

where $c_{min}/\mu < c_H^* < c_{max}$. Then, $B_L(c_H^*, c_L^*) = c_L^*$ and $c_{min} < c_L^* < c_{max}$ follow immediately. Such an equilibrium is stable if after a slight increase (decrease) in $c_H$, and the corresponding increase (decrease) in $c_L = \mu c_H$, the benefit from voting falls short of (exceeds) the cost so that the share of voters falls (rises) back to the equilibrium. Formally, defining $\mathcal{B}_H(c_H) \equiv B_H(c_H, \mu c_H)$ for all $c_H$, we we can write the expected benefit for type $H$ as a function of only the cutoff $c_H$. With this definition, the equilibrium is stable if $\frac{\partial \mathcal{B}_H}{\partial c_H} - 1 < 0$.

In order to show that the subsequent analysis of stable interior equilibria is well founded, we complete this section by proving that such an equilibrium exists for some

[8] More specifically, there are six possible types of first period equilibria: equilibria where (i) everyone votes, (ii) nobody votes, (iii) all high type individuals vote and none of low type individuals votes, (iv) all high type individuals vote and some of low type individuals vote, (v) some of high type individuals vote and none of low type individuals votes, (vi) some of high type individuals and some of low type individuals vote.

parameter values of the model. For this purpose, consider the following inequalities:

$$B_H(c_{min}/\mu, c_{min}) = \alpha_H \left\{ \frac{w_1}{2}\Pi(qF(c_{min}/\mu)) \right. \qquad (8)$$

$$\left. + w_2 qF(c_{min}/\mu)(n-1)[\alpha_H - d] \right\} > c_{min}/\mu$$

$$B_H(c_{max}, \mu c_{max}) = \alpha_H \left\{ \frac{w_1}{2}\Pi(q + (1-q)F(\mu c_{max})) \right. \qquad (9)$$

$$\left. + w_2(n-1)[q(\alpha_H - d) - (1-q)F(\mu c_{max})(d - \alpha_L)] \right\} < c_{max}$$

$$\frac{(1-q)F(\mu c_H)}{qF(c_H)} \leq \frac{\alpha_H - d}{d - \alpha_L} \leq \frac{(1-q)(1-F(\mu c_H))}{q(1-F(c_H))} \qquad (10)$$

Note that inequality (10) is equivalent to the inequalities, $\lambda_H \alpha_H + (1-\lambda_H)\alpha_L \geq d$ and $(1-\lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$, which ensure that only voters match among each other in the second period. Inequalities (8) and (9) are boundary conditions requiring that the benefit of voting exceeds (falls short of) the cost of voting if the turnout is very low (very high).

**Proposition 1.**

*(i) If inequality (10) holds for all $c_H \in [c_{min}/\mu, c_{max}]$, and inequalities (8) and (9) hold, then a stable interior equilibrium with signaling exists.*

*(ii) There exist parameter values of the model which satisfy simultaneously the above inequalities.*

# 4 Comparative Statics

In this section, we provide the main comparative statics of stable interior equilibria with signaling, which have been shown to exist in the previous section. In the first subsection 4.1, we derive some direct effects of the model's parameters on equilibrium turnout. In subsection 4.2, we turn to the interaction between signaling and the incentives to vote, which is the main focus of our analysis.

## 4.1 Direct effects

By substituting the posterior beliefs (4) and (5) in equation (6) and by linking the cut-off value of low types to the cut-off value of high types via $c_L = \mu c_H$, the equilibrium condition (7) can be formulated as:

$$\begin{aligned}\mathcal{B}_H(c_H) &\equiv B_H(c_H, \mu c_H)\\ &= \alpha_H \Big\{ \frac{w_1}{2} \Pi \left[ qF(c_H) + (1-q)F(\mu c_H) \right] \\ &\quad + w_2(n-1)[qF(c_H)(\alpha_H - d) + (1-q)F(\mu c_H)(\alpha_L - d)] \Big\} = c_H \end{aligned} \tag{11}$$

By using the implicit function theorem one can compute the effect of a change of a parameter, say $x$, of the model to the equilibrium cutoff $c_H^*$:

$$\frac{dc_H^*}{dx} = -\frac{\frac{\partial \mathcal{B}_H}{\partial x}}{\frac{\partial \mathcal{B}_H}{\partial c_H} - 1} \tag{12}$$

Since we are considering a stable equilibrium of the game, we know that $(\partial \mathcal{B}_H)/(\partial c_H) < 1$, so that the denominator of the above expression is negative. Therefore, the change of the equilibrium cutoff $c_H^*$ has the same sign as the change of the total expected utility $(\mathcal{B}_H)$ with respect to the parameter $x$ . Also, recall that $p^* = qF(c_H^*) + (1-q)F(\mu c_H^*)$. As a consequence, we have the following comparative statics of the model:

(i) $\frac{dp^*}{dd} < 0$: An increase in the cost of the second stage interaction decreases the value of signaling and equilibrium turnout.

(ii) $\frac{dp^*}{dw_1} > 0$ and $\frac{dp^*}{dw_2} > 0$: Directly increasing the significance that voters put in the election or the significance of signaling increases equilibrium turnout.

(iii) $\frac{dp^*}{dN} < 0$ but $\frac{dp^*}{dn} > 0$: Increasing the size of the electorate reduces the probability of being pivotal and the value of the election and thus equilibrium turnout decreases. This is, of course, a direct implication of the $\Pi(p)$ function, which is the same as in Borgers (2004). However, notice that, even if $N$ is arbitrarily large, the value of signaling remains strictly positive in the set of equilibria that we examine and the equilibrium turnout does not fall to zero. To put it differently, even if we examine arbitrarily large societies, we can find values for the remaining parameters such that an interior equilibrium with strictly positive turnout exists and the **paradox of not voting** does not take place. This is because, even though agents can not affect the outcome of the election, they receive strictly positive utility by signaling their type to other agents. On the other hand, an increase in the number of neighbors increases the value of signaling and equilibrium turnout.

The comparative statics above have a straightforward interpretation, which comes directly from the model: any change that increases the value of the electoral outcome or the value of signaling or both, increases the willingness of the marginal voter to vote and, therefore, it increases the equilibrium turnout. However, as the following result shows, the model also generates some effects which are more involved.

(iv) $\frac{dp^*}{d\alpha_H}$ has an ambiguous sign[9]: On the one hand, turnout ratio of high type agents increases unambiguously. On the other hand, turnout ratio of low type agents may decrease or increase due to the two opposing effects: the decrease in voting benefit due to lower pivotal probability (for a given $c_L$, $c_H = c_L/\mu$ is higher since an increase in $\alpha_H$ decreases $\mu$, which leads to more turnout of high type agents), and the increase in signaling benefit through the increased benefit of a match with a high type agent and the increase of the numbers of matches with high type agents (again due to the higher turnout of high type agents). If turnout ratio of low type agents increases, then overall turnout ratio clearly increases. Otherwise, the result is ambiguous.

## 4.2 Interaction of signaling and voting incentives

After discussing these comparative static effects, which are direct consequences of introducing signaling into the model, we turn to the more subtle, and possibly even more interesting, indirect effects. Specifically, we ask: How do the benefits of voting and of signaling interact? Does the presence of signaling increase the sensitivity of turnout to the importance of the election outcome for voters? In other words, we would like to investigate the conditions under which the presence of signaling in a voting game **reinforces** or **dampens** the sensitivity of turnout to the electoral incentives. This is interesting both in terms of empirical implications (are countries with better connected communities expected to have more volatile turnout?) and in terms of policy implications (should governments adopt community friendly policies to increase the sensitivity of voters to political issues?). For brevity, whenever the sensitivity of the turnout to electoral incentives increases with signaling we say that we have a **reinforcing signaling** effect, while whenever the sensitivity of the turnout to electoral incentives decreases

[9] The computations and more detailed explanations are in the appendix. The other comparative static analyses $\frac{dp^*}{d\alpha_L}$ and $\frac{dp^*}{dq}$ will be skipped to save space, since they have similar flavor to $\frac{dp^*}{d\alpha_H}$.

with signaling we say that we have a **dampening signaling** effect.

Moreover, we investigate whether there can be a **bandwagon effect**, i.e. whether a voter is more likely to vote when there is higher turnout. Note that in the absence of signaling, this is impossible, since higher turnout decreases the pivotal probability of a voter, who is then less likely to vote. In addition, we ask whether an increase in turnout ratio can be substantial in case of a small increase of the election's significance ($w_1$). Note again that this cannot be the case in the absence of signaling, since the effect of $w_1$ is downgraded by small pivotal probabilities.

### 4.2.1 Reinforcing or dampening signaling effects

In terms of formal analysis, we study whether signaling is reinforcing or dampening by examining how the change of $c_H^*$ due to an increase in the significance of the elections is affected by an increase in the value of signaling. Therefore, if $\frac{d^2 c_H^*}{dw_1 dw_2} > 0$ we have reinforcing signaling and if $\frac{d^2 c_H^*}{dw_1 dw_2} < 0$ we have dampening signaling. Since an increase in the equilibrium cut-off value $c_H^*$ always increases the equilibrium turnout $p^*$ for given values of $q$, $\alpha_H$ and $\alpha_L$, examining the effect on $c_H^*$ also gives us the impact on $p^*$. By setting $x = w_1$ and by taking the derivative of (12) with respect to $w_2$ we find:

$$\frac{d^2 c_H^*}{dw_1 dw_2} = \frac{\frac{\partial^2 \mathcal{B}_H}{\partial c_H \partial w_2} \frac{\partial \mathcal{B}_H}{\partial w_1}}{\left(\frac{\partial \mathcal{B}_H}{\partial c_H} - 1\right)^2}$$

Since the denominator and $\frac{\partial \mathcal{B}_H}{\partial w_1}$ are both positive, the sign of the expression above has the same sign as $\frac{\partial^2 \mathcal{B}_H}{\partial c_H \partial w_2}$. By computing the latter cross-derivative and rearranging we find that we have reinforcing signaling if and only if (recall that $\mu = \alpha_L/\alpha_H$):

$$(\alpha_H - d)qf(c_H) + (\alpha_L - d)(1-q)\mu f(\mu c_H) > 0 \qquad (13)$$

Inequality (13) illustrates the interaction of voting and signaling incentives. When the election importance ($w_1$) increases, there are $qf(c_H)$ additional individuals of high type and $(1-q)\mu f(\mu c_H)$ additional individuals of low type who decide to vote. Inequality (13) states that the expected payoff of matching with these additional voters is positive. In this case, the expected signaling benefit of voting increases, which reinforces the increase in turnout due to the higher importance of the election.

Solving inequality (13) for the parameter $d$ we find a critical threshold value (let us call it $\tilde{\alpha}$), such that if $d$ is below this threshold, then we have reinforcing signaling, while if $d$ is above this threshold we have dampening signaling. We summarize this result in the following proposition, which is directly derived from the analysis so far:

**Proposition 2.** *In any stable interior equilibrium with signaling (i.e. agents interact only with voters) we have a reinforcing signaling effect whenever the cost of matching $d$ is below the threshold value $\tilde{\alpha}$ and dampening signaling otherwise, with*

$$\tilde{\alpha} \equiv \frac{\alpha_H q f(c_H) + \alpha_L (1-q)\mu f(\mu c_H)}{q f(c_H) + (1-q)\mu f(\mu c_H)} \tag{14}$$

Note that, if we define $w_2 p(n-1)[\lambda_H \alpha_H + (1-\lambda_H)\alpha_L - d]$ in equation (6) as the signaling benefit of voting, then it is easy to show that:

$$\frac{\partial^2 \mathcal{B}_H}{\partial c_H \partial w_2} = \frac{1}{w_2} \frac{\partial (\text{signaling benefit})}{\partial c_H}$$

Hence, if an increase in the total turnout has a positive effect on the value of signaling, then this implies that signaling has a reinforcing effect on voting. The interpretation is that if the significance of the elections increases ($w_1$ increases) then turnout will increase because the overall expected benefit for voters increases. But whether this effect is larger or smaller than in a society where the signaling benefit is absent (i.e. Borgers (2004)) or where communities are less important (lower value of $w_2$), depends on the impact of the increased turnout on the signaling benefit. If turnout has a positive impact on signaling then the increase in turnout will be greater in the society with stronger community ties ($\frac{d^2 c_H^*}{dw_1 dw_2} > 0$), because the initial increase in the value of voting is further reinforced by the fact that voting is also more beneficial for signaling one's type to his "neighbors". Of course, the opposite is true if the signaling benefit is negatively affected by higher turnout.

Proposition 2 makes clear that in a society where the cost of social interactions is low ($d < \tilde{\alpha}$), for instance due to inadequate substitutes to social interactions or because of well-established communication channels, signaling has a reinforcing effect, while the opposite is true for a society with high cost of social interactions. Hence, we expect the turnout ratio to be more sensitive to the importance of the electoral outcome in societies with low cost of social interactions.

Beyond this general result, it is worthwhile to investigate in more detail whether, and in what circumstances, the condition $d < \tilde{\alpha}$ is likely to be satisfied in an equilibrium with signaling. To answer this question, we relate $\tilde{\alpha}$ to the inequalities laid down in section 3.1: $\lambda_H\alpha_H + (1-\lambda_H)\alpha_L \geq d$ and $(1-\lambda_L)\alpha_H + \lambda_L\alpha_L \leq d$. These inequalities implicitly define an interval $[d_L, d_H]$, within which the cost $d$ of the match must lie for an equilibrium with signaling to obtain.

If $\tilde{\alpha}$ is greater than the upper bound of the interval $[d_L, d_H]$, i.e. $d_H$, then signaling has a reinforcing effect on voting irrespectively of the other parameters of the model. If $\tilde{\alpha}$ is lower than the lower bound of the interval, i.e. $d_L$, then signaling has a dampening effect on voting, irrespectively of the other parameters of the model, and if $\tilde{\alpha}$ is in the interior of the interval, the effect of signaling is either reinforcing or dampening, depending on the other parameters of the model[10]:

**Proposition 3.** *Consider an interior equilibrium with signaling and cutoff value $c_H^*$ for the high types. Then:*

*(i) If $\frac{f(c_H^*)}{F(c_H^*)} > \frac{\mu f(\mu c_H^*)}{F(\mu c_H^*)}$, then the effect of signaling is reinforcing.*

*(ii) If $\frac{f(c_H^*)}{1-F(c_H^*)} > \frac{\mu f(\mu c_H^*)}{1-F(\mu c_H^*)}$, the effect of signaling is reinforcing for some parameter values and dampening for the rest.*

*(iii) If $\frac{f(c_H^*)}{1-F(c_H^*)} < \frac{\mu f(\mu c_H^*)}{1-F(\mu c_H^*)}$, then the effect of signaling is dampening.*

Note that the condition of part (ii) in Proposition 3 is a weaker version of the increasing hazard rate, which is commonly used in the literature. This means that, if the distribution of voting costs satisfies the increasing hazard rate property, then whether signaling has a reinforcing or dampening effect depends on the cost of social interactions, $d$, as given in Proposition 2. On the other hand, ensuring that all the stable interior equilibria of the model for any set of parameter values exhibit reinforcing signaling requires the condition of part (i). This condition, which is akin to the "reverse" hazard rate, is stronger than condition (ii). The most commonly used distributions in the literature, such as the uniform, the normal and the exponential distribution, do not satisfy the condition of part (i) but satisfy the condition of part (ii) globally.

[10] The proof is included in the appendix.

#### 4.2.2 Bandwagon Effect

Next, we investigate whether there can be a bandwagon effect in our model. Mathematically, a bandwagon effect exists if and only if[11] $\frac{\partial \mathcal{B}_H}{\partial c_H} > 0$, i.e. higher turnout increases the voting benefit of a voter. $\frac{\partial \mathcal{B}_H}{\partial c_H}$ is given by:

$$\begin{aligned}\frac{\partial \mathcal{B}_H}{\partial c_H} =& \alpha_H \Big\{ \frac{w_1}{2} \Pi'(p) \left[ q f(c_H) + (1-q)\mu f(\mu c_H) \right] \\ &+ w_2(n-1)[q f(c_H)(\alpha_H - d) + (1-q)\mu f(\mu c_H)(\alpha_L - d)] \Big\}\end{aligned} \tag{15}$$

Since $\Pi'(p)$ is negative, the first term in the curly brackets is negative. This term shows that electoral benefit decreases with higher turnout. The second term, which corresponds to the change of signaling benefit, is positive if and only if signaling is reinforcing, i.e. inequality (13) holds. Hence, a necessary condition for a bandwagon effect ($\frac{\partial \mathcal{B}_H}{\partial c_H} > 0$) is reinforcing signaling. Given that signaling is reinforcing, a bandwagon effect exists as long as the second term is higher in absolute value than the first term, for instance, for a high enough value for the importance of social interactions ($w_2$).

The intuition is as follows: With a higher turnout, electoral benefit of a voter decreases due to a smaller pivotal probability. However, if signaling benefit increases with a higher turnout, or equivalently if signaling is reinforcing, then the bandwagon effect may arise. The bandwagon effect exists when the increase in signaling benefit is higher in magnitude than the decrease in electoral benefit.

#### 4.2.3 Magnitude of $dc_H^*/dw_1$

Until here, we were interested in the sign of various effects. Finally, we analyze the magnitude of the increase of turnout ratio due to a small increase of the election's significance ($w_1$). Note that in a model of voting which does not include signaling benefit, the response of turnout to changes of $w_1$ is small due to low pivotal probabilities for voters. Therefore, it is important to see whether the inclusion of the signaling benefit can change this result.

As we showed earlier, the election's significance becomes more important for turnout ratio when signaling is reinforcing. Indeed, if this reinforcement is strong enough so that there exists an important bandwagon effect, a small change in the importance of

[11]Expressing this condition in terms of the voting benefit of a high type agent is sufficient, since the voting benefit of a low type agent is proportional.

the election may have a large impact on equilibrium turnout. Mathematically, replacing $x$ by $w_1$ in equation (12) gives:

$$\frac{dc_H^*}{dw_1} = -\frac{\frac{\partial \mathcal{B}_H}{\partial w_1}}{\frac{\partial \mathcal{B}_H}{\partial c_H} - 1} \tag{16}$$

where $\frac{\partial \mathcal{B}_H}{\partial c_H}$ is given in equation (15) and $\frac{\partial \mathcal{B}_H}{\partial w_1}$ is given by

$$\frac{\partial \mathcal{B}_H}{\partial w_1} = \frac{\alpha_H}{2}\Pi(p)$$

Since $\Pi(p)$ is relatively small, the numerator in equation (16) is expected to be small. In the absence of signaling benefit ($w_2 = 0$), the denominator in absolute value is higher than 1, since $\frac{\partial \mathcal{B}_H}{\partial c_H}$ is negative. This leads to a low magnitude of $\frac{dc_H^*}{dw_1}$. However, in the presence of signaling, if signaling is reinforcing, $\frac{\partial \mathcal{B}_H}{\partial c_H}$ can be arbitrarily close to 1 (a stable equilibrium implies that $\frac{\partial \mathcal{B}_H}{\partial c_H} < 1$) for some high values of $w_2$. Then, this leads to a small denominator in absolute value and therefore to an important magnitude of $\frac{dc_H^*}{dw_1}$.

The intuition behind this result is that, if social interactions are very important for voters (high $w_2$), then even a small increase in the importance of the election may generate a large increase in turnout, because of the importance of signaling effects. In other words, since voters expect other voters to turn out in higher numbers, their own incentive to vote increases significantly due to signaling purposes and this may generate a substantial increase on total turnout. This is an important result of our paper, because it depends crucially on the existence of signaling benefits and can not be generated by the existing literature on rational voting.

# 5 Conclusion

The paper presents a formal model of voting as signaling device. By observing the voting behavior of others in their social circle, voters receive a signal about their 'neighbor's' value in social interactions. This generates an additional incentive to vote, apart from affecting the outcome of the election, as the early rational voting theory predicts. This additional incentive can account for the paradox of not voting in large societies and the role of social pressures in electoral turnouts. Moreover, the model generates several predictions which are consistent with empirical findings.

We believe that the model can be extended in order to shed light on the interaction between voting incentives and the role of political parties. In our model, party preferences are assumed to be independently distributed in each neighborhood. Also the benefit of social interaction is assumed to be independent of voters' preferences over political parties. However, one would reasonably assume that cooperation among individuals of similar ideological position is more beneficial than if they have very dissimilar views. Relaxing these assumptions may lead to understand better political parties' strategic use of advertising and the role of party activists, depending on the characteristics of neighborhoods.

Overall, we believe that this is a very fruitful avenue for further research and we intend to extend our model in the near future in these directions.

# Appendix

## First-Stage Equilibrium

### Proof of Proposition 1

When we plot $\mathcal{B}_H(c_H)$ on $c_{min}/\mu < c_H < c_{max}$, the intersection $c_H^*$ with the 45° line would be an interior equilibrium satisfying (7). By the continuity of $\mathcal{B}_H(c_H)$ on the interval $[c_{min}/\mu, c_{max}]$, if $\mathcal{B}_H(c_{min}/\mu) > c_{min}/\mu$ (i.e. the starting point is above the 45° line) and $\mathcal{B}_H(c_{max}) < c_{max}$ (i.e. the ending point is below the 45° line), then at least one such intersection exists. Moreover, since at least one intersection is such that $\mathcal{B}_H(c_H)$ cuts the 45° line from above, a stable interior equilibrium exists if these two conditions are satisfied. In an interior equilibrium with signaling, the second period benefit is $w_2 p(n-1)[\lambda_H \alpha_H + (1-\lambda_H)\alpha_L - d]$. With the cutoff points $c_H = c_{min}/\mu$ and $c_L = c_{min}$, $p$ is equal to $p = qF(c_{min}/\mu)$ and $\lambda_H$ is equal to $\lambda_H = \frac{qF(c_{min}/\mu)}{qF(c_{min}/\mu)} = 1$. With the cutoff points $c_H = c_{max}$ and $c_L = \mu c_{max}$, $p$ is equal to $p = q + (1-q)F(\mu c_{max})$ and $\lambda_H$ is equal to $\lambda_H = \frac{q}{q+(1-q)F(\mu c_{max})}$. Replacing $p$ and $\lambda_H$ in (6) and rearranging, one finds that $\mathcal{B}_H(c_{min}/\mu) > c_{min}/\mu$ and $\mathcal{B}_H(c_{max}) < c_{max}$ are equivalent to inequalities (8) and (9).

In addition, we have to make sure that this intersection $c_H^*$ gives an equilibrium with signaling. This is the case if the two conditions $\lambda_H \alpha_H + (1-\lambda_H)\alpha_L \geq d$ and $(1-\lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$ hold for all $c_H \in [c_{min}/\mu, c_{max}]$ (i.e. for all possible intersection points). These two conditions are equivalent to inequality (10) holding for all $c_H \in [c_{min}/\mu, c_{max}]$.

(ii) The lhs of inequality (8) is always positive. Hence, this inequality is satisfied for low enough $c_{min}$. The lhs of inequality (9) is bounded above by $\alpha_H \big\{ \frac{w_1}{2} + w_2(n-1)q(\alpha_H - d) \big\}$. Hence, this inequality is satisfied for high enough $c_{max}$.

The lhs of inequality (10) is lower than $(1-q)/q$ since $F(\mu c_H) < F(c_H)$ for all $c_H \in [c_{min}/\mu, c_{max}]$. Similarly, the rhs of inequality (10) is greater than $(1-q)/q$ since $1 - F(\mu c_H) > 1 - F(c_H)$ for all $c_H \in [c_{min}/\mu, c_{max}]$. Then, for instance, if $d$ is such that $\frac{\alpha_H - d}{d - \alpha_L} = \frac{1-q}{q}$ (equivalently $q\alpha_H + (1-q)\alpha_L = d$), both conditions are satisfied. Hence, there is a neighborhood of values of $d$ around $q\alpha_H + (1-q)\alpha_L$ in which both conditions are satisfied. Note that this neighborhood for $d$ is consistent with the fact that inequalities (8) and (9) hold for some parameter values, since the latter inequalities are satisfied by appropriate choice of $c_{min}$ and $c_{max}$, irrespective of $d$. ■

## Comparative Statics

**Claim:** $\frac{dp^*}{d\alpha_H}$ *has an ambiguous sign.*

**Proof:** $\frac{\partial \mathcal{B}_H}{\partial \alpha_H}$ is given by

$$\begin{aligned}\frac{\partial \mathcal{B}_H}{\partial \alpha_H} =& \Big\{\frac{w_1}{2}\Pi(p) + w_2(n-1)[qF(c_H)(\alpha_H - d) + (1-q)F(\mu c_H)(\alpha_L - d)]\Big\} + \\ & \alpha_H\Big\{\frac{w_1}{2}\Pi'(p)(1-q)f(\mu c_H)(-\frac{\alpha_L}{\alpha_H^2})c_H \\ & + w_2(n-1)\big[qF(c_H) + (1-q)f(\mu c_H)(\alpha_L - d)(-\frac{\alpha_L}{\alpha_H^2})c_H\big]\Big\} \end{aligned} \tag{17}$$

The term in the first bracket is clearly positive and corresponds to the direct effect of the increased benefit of voting and signaling. The first and the second terms in the second bracket are also positive and correspond respectively to the increase in voting benefit through higher pivotal probability (for a given $c_H$, $c_L = \mu c_H$ is lower since an increase in $\alpha_H$ decreases $\mu$, which leads to less turnout of low type agents), and the increase in signaling benefit through the increased benefit of a match with a high type agent and the decrease of the numbers of matches with low type agents (again due to the lower turnout of low type agents). Hence, turnout ratio of high type agents increases unambiguously.

Similar to $\mathcal{B}_H(c_H)$, we define $\mathcal{B}_L(c_L)$ as

$$\begin{aligned}\mathcal{B}_L(c_L) &\equiv B_L(c_L/\mu, c_L) \\ &= \alpha_L\Big\{\frac{w_1}{2}\Pi\left[qF(c_L/\mu) + (1-q)F(c_L)\right] \\ &+ w_2(n-1)\big[qF(c_L/\mu)(\alpha_H - d) + (1-q)F(c_L)(\alpha_L - d)\big]\Big\} = c_L \end{aligned} \tag{18}$$

By the same argument as in page 13, $dc_L/d\alpha_H$ has the same sign as $\partial \mathcal{B}_L/\partial \alpha_H$ which is given by

$$\frac{\partial \mathcal{B}_L}{\partial \alpha_H} = \alpha_L\Big\{\frac{w_1}{2}\Pi'(p)qf(c_L/\mu)\frac{c_L}{\alpha_L} + w_2(n-1)\big[qF(c_L/\mu) + qf(c_L/\mu)(\alpha_H - d)\frac{c_L}{\alpha_L}\big]\Big\} \tag{19}$$

The first term in the bracket is negative and corresponds to the first effect mentioned above, whereas the second term is positive and corresponds to the second effect. Hence, the sign of the change of low type agents' turnout ratio is ambiguous. Therefore, overall turnout ratio can increase or decrease, depending on parameter values. ■

**Proof of Proposition 3:** First we derive the thresholds $d_H$ and $d_L$ from section 3.1, by substituting the relevant values for $\lambda_H$ and $\lambda_L$ into $\lambda_H\alpha_H + (1-\lambda_H)\alpha_L \geq d$ and $(1-\lambda_L)\alpha_H + \lambda_L\alpha_L \leq d$:

$$d_H = \lambda_H\alpha_H+(1-\lambda_H)\alpha_L \Rightarrow d_H = \tfrac{qF(c_H)}{qF(c_H)+(1-q)F(\mu c_H)}\alpha_H+\left(1-\tfrac{qF(c_H)}{qF(c_H)+(1-q)F(\mu c_H)}\right)\alpha_L \Rightarrow$$

$d_H = \frac{\alpha_H qF(c_H)+\alpha_L(1-q)F(\mu c_H)}{qF(c_H)+(1-q)F(\mu c_H)}$

Similarly:

$d_L = (1-\lambda_L)\alpha_H + \lambda_L\alpha_L \Rightarrow d_L = \frac{\alpha_H q(1-F(c_H))+\alpha_L(1-q)(1-F(\mu c_H))}{q(1-F(c_H))+(1-q)(1-F(\mu c_H))}$

For part (i), suppose that $\frac{f(c_H)}{F(c_H)} \geq \frac{\mu f(\mu c_H)}{F(\mu c_H)}$.

$$\frac{f(c_H)}{F(c_H)} \geq \frac{\mu f(\mu c_H)}{F(\mu c_H)} \Leftrightarrow (\alpha_H-\alpha_L)f(c_H)F(\mu c_H) \geq (\alpha_H-\alpha_L)\mu f(\mu c_H)F(c_H) \Leftrightarrow$$

$$\alpha_H f(c_H)F(\mu c_H) + \alpha_L\mu f(\mu c_H)F(c_H) \geq \alpha_H\mu f(\mu c_H)F(c_H) + \alpha_L f(c_H)F(\mu c_H)$$

Multiplying both sides by $q(1-q)$ and adding $\alpha_H q^2 f(c_H)F(c_H)$ and $\alpha_L(1-q)^2\mu f(\mu c_H)F(\mu c_H)$ on both sides yields:

$$\alpha_H q^2 f(c_H)F(c_H)+\alpha_H q(1-q)f(c_H)F(\mu c_H)+\alpha_L q(1-q)\mu f(\mu c_H)F(c_H)+\alpha_L(1-q)^2\mu f(\mu c_H)F(\mu c_H) \geq$$
$$\alpha_H q^2 f(c_H)F(c_H)+\alpha_H q(1-q)\mu f(\mu c_H)F(c_H)+\alpha_L q(1-q)f(c_H)F(\mu c_H)+\alpha_L(1-q)^2\mu f(\mu c_H)F(\mu c_H) \Leftrightarrow$$

$$[\alpha_H qf(c_H)+\alpha_L(1-q)\mu f(\mu c_H)][qF(c_H)+(1-q)F(\mu c_H)] \geq$$
$$[\alpha_H qF(c_H)+\alpha_L(1-q)F(\mu c_H)][qf(c_H)+(1-q)\mu f(\mu c_H)] \Leftrightarrow$$

$$\frac{\alpha_H qf(c_H)+\alpha_L(1-q)\mu f(\mu c_H)}{qf(c_H)+(1-q)\mu f(\mu c_H)} \geq \frac{\alpha_H qF(c_H)+\alpha_L(1-q)F(\mu c_H)}{qF(c_H)+(1-q)F(\mu c_H)} \Leftrightarrow \tilde{\alpha} \geq d_H$$

From the lines above, we conclude more specifically that

$$\frac{f(c_H)}{F(c_H)} \quad > (=) \quad \frac{\mu f(\mu c_H)}{F(\mu c_H)} \Leftrightarrow \tilde{\alpha} \quad > (=) \quad d_H$$

When $\tilde{\alpha}$ is greater than (resp. equal to) $d_H$, this implies that any value of $d$ that satisfies the equilibrium conditions also satisfies $d < \tilde{\alpha}$ (resp. $d \leq \tilde{\alpha}$) . Hence $\frac{d^2 c_H^*}{dw_1 dw_2} > 0$ (resp. $\geq 0$).

For part (ii), substitute in the proof above the terms $1 - F(c_H)$ and $1 - F(\mu c_H)$ for the terms $F(c_H)$ and $F(\mu c_H)$ respectively and iterate the same steps. Then we obtain:

$$\frac{f(c_H)}{1 - F(c_H)} > (=) \frac{\mu f(\mu c_H)}{1 - F(\mu c_H)} \Leftrightarrow$$

$$\frac{\alpha_H q f(c_H) + \alpha_L (1-q) \mu f(\mu c_H)}{q f(c_H) + (1-q) \mu f(\mu c_H)} > (=) \frac{\alpha_H q (1 - F(c_H)) + \alpha_L (1-q)(1 - F(\mu c_H))}{q(1 - F(c_H)) + (1-q)(1 - F(\mu c_H))} \Leftrightarrow$$

$$\tilde{\alpha} > (=) d_L$$

When $\tilde{\alpha}$ is greater than $d_L$, the cost of the match $d$ may satisfy $d < \tilde{\alpha}$ or not. This depends on the other parameters of the model. Hence, either $\frac{d^2 c_H^*}{dw_1 dw_2} \geq 0$ or $\frac{d^2 c_H^*}{dw_1 dw_2} < 0$. When $\tilde{\alpha}$ is equal to $d_L$, $d \geq \tilde{\alpha}$. Hence, $\frac{d^2 c_H^*}{dw_1 dw_2} \leq 0$.

Finally, part (iii) follows from part (ii). This is because when the initial condition of part (ii) does not hold, then $\tilde{\alpha} < d_L$, which implies that the condition $d \leq \tilde{\alpha}$ is mutually exclusive with the equilibrium conditions. ■

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