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**A Simple Approach to Specifying the Weights  
of the HDI Index**

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# **A Simple Approach to Specifying the Weights of the HDI Index**

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## **Abstract:**

Starting from a representative welfare function, this paper developed a simple method to endogenously specify the weights for the Human Development Index (HDI), and finds that the current equal-weighted HDI significantly biases down the weight of life expectancy. The weights proposed by this study may more properly reflect the human-centered development of the HDI.

**JEL: O15, D6**

**Key Words:** HDI, Weights, Welfare Function, Ranking

**Running title: Weights of HDI Index**

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Starting from a representative welfare function, this paper developed a simple method to endogenously specify the weights for the Human Development Index (HDI), and finds that the current equal-weighted HDI significantly biases down the weight of life expectancy. The weights proposed by this study may more properly reflect the human-centered development of the HDI.

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### **Introduction**

Since the initiation of the Human Development Index in 1990, there have been many criticisms on choice of the dimensions and the related weights as well (Sagara and Najam 1998). In particular, the choices of the weights are critical for the robustness of the ranking (Foster, McGillivray and Seth 2009). There are many approaches to specify the weights, such as the Data-Driven Approach, the Normative Approach and the Hybrid Approach. Decanq and Ana LUGO (2010) have a comprehensive review for the methods of weight choices.

Most of the approaches do not have solid theoretical foundations. Then United Nations Development Program (UNDP) simply chooses the geometric mean of the three normalized indices---life expectancy, education and income in its 2010 report (UNDP

2010) which obviously is not convincing as it neither has a solid theoretical foundation, nor can it identify the different importance between the three indices. Based on a representative welfare function, we try to set up a theoretical foundation for choosing the weights for HDI, which is also applied to the analysis of 2010 HDI.

### Model

We start from a representative welfare function and assume it is a Cobb-Douglas form,

$$\ln V_t = \sum_i w_i \ln x_{it} \quad (1)$$

where  $V_t$  is the welfare measuring human development at time  $t$  and  $x_{it}$  is a component  $i$  affecting the welfare; and  $w_i$  is the distribution parameter. If the welfare function is homogeneous of degree one, that is  $\sum_i w_i = 1$ , then  $w_i$  can be looked as the weight for the components.

Taking differentiation with respect to  $t$  in both sides of equation (1) shows

$$\frac{d \ln V_t}{dt} = \sum_i w_i \frac{d \ln x_{it}}{dt} \quad (2)$$

Given the nature of strong separation for the Cobb-Douglas style utility function, if keeping the growth rate of the welfare constant, such that  $\frac{d \ln V_t}{dt} \equiv M$ , there is a trade-off between the different components. For instance, if only a factor  $i$  changes, and other factors keep constant, so that

$$M = w_i \frac{d \ln x_{it}}{dt} \quad (3)$$

Equation (3) can be rewritten as

$$w_i = M / \left( \frac{d \ln x_{it}}{dt} \right) \quad (4)$$

Here  $\frac{d \ln x_{it}}{dt}$  is the growth rate of the factor  $i$ , which can be easily estimated by

some econometric models as shown below. In order to satisfy the condition of homogenous degree of one, we should scale function (4), and

$\sum_j w_j = M \sum_j [1 / (\frac{d \ln x_{jt}}{dt})] = 1$ , which leads to

$$M = 1 / \sum_j [1 / (\frac{d \ln x_{jt}}{dt})] \quad (5)$$

Substitution of equation (5) in to (4) yields,

$$w_i = 1 / (\frac{d \ln x_{it}}{dt}) / \sum_j [1 / (\frac{d \ln x_{jt}}{dt})] \quad (6)$$

Equation (6) is the weight which can be used for the Human Development Index (HDI). Now we can estimate  $\frac{d \ln x_{it}}{dt}$ , the growth rate of the factor  $i$ , by the specification

$$\ln x_{it} = \ln x_{io} + a_i t \quad (7)$$

Where  $d \ln x_{it} / dt = a_i$ . Rewriting Equation (6) gives,

$$w_i = \frac{\frac{1}{a_i}}{\sum_j \frac{1}{a_j}} \quad (8)$$

Here  $w_i$  can be used as the weights in the human development indicators, or generally in multidimensional Indices of well-being.

## Discussion

Employing the data from UNDP with 194 countries or regions from 2000 through 2010, we estimate equation (7) by the fixed-effects models respectively for life expectancy, education years and income which are the three components of the HDI indices. The estimation results are reported in Table 1. We find they fit the data very well.

By equation (7), the weights for life expectancy, education years and income can be calculated respectively as 0.669, 0.243 and 0.089. It indicates that the current equal weights used by the UNDP significantly bias down the weight of life expectancy and bias up those of education years and income. We find that people are more careful about their health, while income is not so important. The results may more properly reflect the purpose of the HDI---“to shift the focus of development economics from national income accounting to people centered policies” (Haq et al. 1995).

Using the weights, we can re-compute the HDI. We reported the top 20 countries in Table 2. For comparison, the top 20 countries in the 2010 UNDP report with equal weights are also reported. Comparing the two results, there are a few significant changes. First, the top two in this study are still Australia and Norway, but the order

changed. Second, a few countries with long life expectancy perform well in this ranking. For instance, Japan and Iceland are ranked No. 4 and 5 which respectively are No. 11 and 17 in the UNDP. The countries without good performance in the new ranking include the United States and Liechtenstein which drop from No. 4 and 6 to No.12 and 23 respectively.

## **Conclusion**

There are many arguments regarding the choice of the weights for the multidimensional indices of well-being, such as Human Development Index. This paper starts from a representative welfare function and derives a simple method to endogenously specify the weights for the HDI, and finds that current HDI significantly biases down the weight of life expectancy. The weights in this study may more properly reflect the human-centered development of the HDI.

## **References**

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Table 1, Estimation Results

	Log( Life Expectancy)		Log( Education Years)		Log (GNI per Capita)	
	Coefficient	S.D.	Coefficient	S.D.	Coefficient	S.D.
Year ( $a_i$ )	0.0039	0.0001**	0.0109	0.0003**	0.0298	0.0008**
Intercept	4.1823	0.0006**	2.1345	0.0021**	8.4966	0.0052**
$w_i$	0.669		0.243		0.089	

\*\* denotes the significant level of 0.1%.



Table 2, Top 20 countries of the HDI in 2010

Rank	This Study		UNDP	
	Countries	HDI	Countries	HDI
1	Australia	0.971	Norway	0.938
2	Norway	0.955	Australia	0.937
3	New Zealand	0.951	New Zealand	0.907
4	Japan	0.942	United States	0.902
5	Iceland	0.934	Ireland	0.895
6	Ireland	0.933	Liechtenstein	0.891
7	Sweden	0.929	Netherlands	0.890
8	Canada	0.928	Canada	0.888
9	Israel	0.926	Sweden	0.885
10	Switzerland	0.925	Germany	0.885
11	Netherlands	0.924	Japan	0.884
12	United States	0.924	Korea (Republic of)	0.877
13	France	0.923	Switzerland	0.874
14	Germany	0.923	France	0.872
15	Korea (Republic of)	0.919	Israel	0.872
16	Spain	0.918	Finland	0.871
17	Hong Kong, China (SAR)	0.914	Iceland	0.869
18	Finland	0.912	Belgium	0.867
19	Italy	0.912	Denmark	0.866
20	Belgium	0.910	Spain	0.863

Note: Liechtenstein is ranked No. 23 in this study.