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Distributional Justice and Efficiency: Integrating Inequality Within and Between Dimensions in Additive Poverty Indices

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Distributional Justice and Efficiency: Integrating Inequality Within and Between Dimensions in Additive Poverty Indices

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Abstract

According to Sen (1976), any reasonable poverty index ought to be sensitive to inequality. In a multidimensional framework, inequality between poverty dimensions is traditionally treated as association sensitivity. Such an approach, however, is based exclusively on efficiency considerations, thereby neglecting all aspects of distributive justice. This paper introduces a new property for dealing with inequality that accounts for both efficiency as well as distributive justice. Based on the new property, it then proceeds to derive a new class of inequality-sensitive poverty measures whose advantages are demonstrated by an empirical application to 28 developing countries.

JEL Classification: I32

Keywords: Multidimensional poverty measurement, counting indices, inequality, correlation sensitivity, identification

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Introduction

The fact that poverty is a multidimensional phenomenon is undisputed, even in the income poverty literature. In fact, income is not supposed to be important per se but rather to serve as an indicator for economic resources that enable individuals to satisfy their multidimensional needs. However, the suitability of insufficient income as indicator for poverty has been increasingly questioned (e.g. Rawls, 1971; Sen, 1985; Drèze and Sen, 1989; UNDP, 1995). This paper utilises a multidimensional approach to measure poverty, building on Amartya Sen's capability approach (Sen, 1985; 1992; 1997; 2009). The poor within a society are defined as those unable to achieve a minimum capability set of elementary functionings, like the ability to be well-nourished or to have access to education. Such a definition implies that the opportunity of the poor to develop their human capital and reach their full potential is limited by external circumstances, such as socio-economic background, race, gender, religion, poor health, or malnutrition.

A direct consequence of this definition of poverty is that inequality among the poor is inequality in the inability to develop the own potential due to external circumstances, i.e. factors that are "*beyond the scope of individual responsibility*" (Marrero and Rodríguez, 2010, p.3). This explains why Amartya Sen (1976) so forcefully requires any reasonable poverty index to be sensitive to inequality: inequality among the poor is in fact inequality of opportunity that should be fought not only from a distributive justice perspective, but as well from an aggregate perspective as it wastes human capital and thus limits the overall expansion of capabilities in a society (Sen, 1992; Marrero and Rodríguez, 2010).

In a multidimensional framework, inequality persists in two forms: inequality *within* dimensions (Kolm, 1977) and inequality *between* dimensions (Atkinson and Bourguignon, 1982). Whereas the former is defined as the spread of distributions within poverty dimensions; the latter is usually treated as association sensitivity (e.g. Bourguignon and Chakravarty, 2003; Seth, 2011).

This paper claims that the latter approach is inappropriate as it equates the two concepts distributive justice and efficiency which in reality are in tension, sometimes even opposed to one another. Inequality should not be reduced to an evaluation of how efficient poverty attributes are distributed among the poor but also consider who gains and who loses from redistributions. Therefore, the new property “Inequality Sensitivity (IS)” is introduced that basically requires poverty to increase (in the case of substitutes) or to decrease (in the case of complements) if an association increasing switch between two poor persons comes at the expense of the poorer of the two.[2] It is demonstrated that the new axiom uniquely characterises a class of poverty indices that is actually the first that though additive is nevertheless able to account for both inequality within and between dimensions.

The empirical implications are demonstrated for a sample of 28 developing countries for which three different indices are calculated: i) the M_0 of the Alkire and Foster class of indices (2011) that is insensitive to either type of inequality, ii) the multidimensional FGT class of indices that is sensitive to inequality within dimensions, and, finally, iii) the new class of inequality sensitive poverty indices P_{IS} that, as the name suggests, is sensitive to within and between dimensional inequality. The relevance of the sensitivity requirement with regard to both types of inequality is easily established once the distinct changes in country rankings induced by the switch from one index to the next are investigated.

The paper proceeds as follows. The second section provides a brief introduction in the theoretical background of the paper. Section three lays the axiomatic foundation for the derivation and decomposition of the new class of indices in section four that are utilised in the

[2] I follow the *Auspitz-Lieben-Edgeworth-Pareto* (ALEP) definition of substitutability and complementarity. The ALEP definition considers two attributes to be substitutes if their second cross partial derivatives are positive. Intuitively, an increase in one attribute decreases poverty the less the higher the achievements in the second attribute. In the same way, attributes are considered to be complements, when the respective cross partial derivatives are negative and independent in case they are zero.

empirical application presented in section five. Section six concludes. Throughout the paper, proofs are relegated to the appendix.

Theoretical Background

Let $\mathbb{R}^k$ denote the Euclidean *k*-space, and $\mathbb{R}^k_+ \subset \mathbb{R}^k$ the non-negative *k*-space. Further, let $\mathbb{N}$ denote the set of positive integers. $\mathbf{N} = \{1,...,n\} \subset \mathbb{N}$ represents the set of *n* individuals of a typical society and $\mathbf{D} = \{2,...,d\} \subset \mathbb{N}$ the set of *d* poverty dimensions captured by a set of *k* poverty attributes $\mathbf{K} = \{2,...,k\} \subset \mathbb{N}$.

Let $\mathbf{a} \in \mathbb{R}^{\mathbf{K}}_+$ denote the weight vector for the different attributes with $\sum_{j=1}^{k} a_j = 1$. In the following, I will refer to the quantity of an attribute with which an individual is endowed as an achievement. The achievement vector of individual *i* is represented by $\mathbf{x}_{i\cdot} = (x_{i1},...,x_{ik})$ and the respective achievement matrix of a society with *n* individuals by $\mathbf{X} \in \mathbb{R}^{\mathbf{NK}}_+$ where the *ij*th entry represents the achievement x_{ij} of individual *i* in attribute *j*. Let $\mathcal{X}_n$ be the set of possible achievement matrices of population size *n* and $\mathcal{X}=\cup_{\mathbf{N} \subset \mathbb{N}} \mathcal{X}_n$ the set of all possible achievement matrices. Let z_j denote the poverty threshold of attribute *j* so that individual *i* is deprived in *j* whenever the respective achievement falls short of the threshold level, i.e. whenever $x_{ij} < z_j$. Further, let $\mathbf{z} \in \mathbb{R}^{\mathbf{K}}_{++}$ represent the vector of poverty thresholds chosen for the different attributes, with the *j*th element being z_j, and $\mathbf{Z}$ being the set of all possible vectors of poverty thresholds.

In the context of this paper, a poverty index is a function P: $\mathcal{X} \times \mathbf{Z} \to \mathbb{R}$. For any poverty threshold vector $\mathbf{z} \in \mathbf{Z}$, society $\mathcal{A}$ has a higher poverty level than society $\mathcal{B}$ if and only if $P(\mathbf{X}^{\mathcal{A}};\mathbf{z}) \geq P(\mathbf{X}^{\mathcal{B}};\mathbf{z})$ for any $\mathbf{X}^{\mathcal{A}}, \mathbf{X}^{\mathcal{B}} \in \mathcal{X}$.

Let $\mathbf{c}_i = (c_{i1},...,c_{ik})$ represent the *deprivation vector* of individual *i* such that $c_{ij} = 1$ if $x_{ij} < z_j$ and $c_{ij} = 0$ if $x_{ij} \geq z_j$. Further, let $S_j(\mathbf{X})$ – or simply S_j – denote the set of individuals who are poor with respect to attribute *j* and *q* the overall number of poor individuals in a society. For reasons of simplicity, let $\delta_i = \sum_{j\in\{1,...,k\}:c_{ij}=1} a_j$ denote the sum of weighted deprivations suffered by individual *i*, with $\mu(\boldsymbol{\delta}) = 1/q\sum_{i\in S_j}\delta_i$. Also, let $g_{ij} = (1 - x_{ij}/z_j)$ denote the poverty gap of individual *i* and attribute *j*, with $\mu_j(\mathbf{g}) = 1/q_j\sum_{i\in S_j} g_{ij}$.

Finally, let $\rho: \mathbb{R}^{\mathbf{K}}_{+} \times \mathbb{R}^{\mathbf{K}}_{++} \rightarrow \{0,1\}$ represent an *identification function* according to the *component poverty line approach* so that individual *i* is poor if $\rho(\mathbf{c}_i;\mathbf{z}) = 1$ and not poor if $\rho(\mathbf{c}_i;\mathbf{z}) = 0$. The approach is theoretically founded in the *strong focus axiom* considering each poverty attribute as essential in the sense that compensation is impossible.[3]

Three specifications of the identification function have been suggested so far. The *union method* is based on the assumption that all attributes are *perfect complements* and thus that every deprived person is considered poor. The *intersection method* considers all attributes to be *perfect substitutes* and thus identifies only those individuals as poor who are deprived in every single attribute. Both approaches are extreme cases, repeatedly yielding poverty rates that are plainly inapplicable, being either far too high or far too low (Bérenger and Bresson, 2010; Alkire and Foster, 2011). The third identification method, the *intermediate method*, has been developed as a loophole, considering only those individuals as poor that are deprived in some pre-determined minimum level of weighted deprivations, i.e.

[3] The other main method for the identification of the poor is called *aggregate poverty line approach*. The special feature of this method is that it allows compensation between attributes below and above threshold levels among those who are poor (*Weak Focus Axiom*).

$\rho_{IM}(\mathbf{c}_i;\mathbf{z}) = \begin{cases} 1 & if \quad \delta_i \geq \delta_{IM}^{\min} \\ 0 & if \quad \delta_i < \delta_{IM}^{\min} \end{cases}$ (Mack and Lindsay, 1985; Foster, 2009; Alkire and Foster, 2011). Please note that the intermediate method comprises union and intersection method as extreme cases, i.e. in case $\delta_{IM}^{\min} \mathrel{\hat{=}} \max\{\mathbf{c}_{i.}\} = 1$ and $\delta_{IM}^{\min} \mathrel{\hat{=}} \min\{\mathbf{c}_{i.}\} = 1$, respectively.

Though the intermediate method is a convenient way out of the dilemma of extreme poverty rates, its theoretical justification is questionable. Apart from the fact that the choice of $\delta_{IM}^{\min}$ is arbitrary, the whole method is based on the indirect assumption that up to $\delta_{IM}^{\min}$ attributes are perfect substitutes whereas they are considered perfect complements from $\delta_{IM}^{\min}$ onwards. In response, Rippin (2012) introduced a new identification method that leads to applicable poverty rates and is theoretically founded in the concept of inequality between dimensions. The new identification method is based on a multi- instead of a single step identification function: $\rho_{CS}(\mathbf{c}_{i\cdot};\mathbf{z}) = \begin{cases} h(\mathbf{c}_{i\cdot}) & if \quad \max\{\mathbf{c}_{i\cdot}\} = 1 \\ 0 & if \quad \max\{\mathbf{c}_{i\cdot}\} = 0 \end{cases}$, $h: \mathbb{R}_+^{\mathbf{K}} \rightarrow [0,1]$ being a function of poverty severity that is nondecreasing with a nondecreasing (nonincreasing) marginal[4] in case attributes are substitutes (complements).

[Figure 1]

Instead of differentiating between the poor and the non-poor, the new function differentiates between the non-poor on one hand and different degrees of poverty severity on the other. This way, it accounts for possible association sensitivity among attributes through the specific shape of the function: while it is always nondecreasing in the number of deprivations, the marginal increase in poverty severity is the less the higher the substitutability between attributes.

[4] A function $f(x)$ has a nondecreasing marginal if $f(x_g+1) - f(x_g) \geq f(x_h+1) - f(x_h)$ whenever $x_g \geq x_h$.

The Axiomatic Foundation

Four main aggregation methods have been developed in order to derive a composite index from individual poverty characteristics: i) the *fuzzy set approach*, ii) the *distance function approach*, iii) the *information theory approach*, and iv) the *axiomatic approach* (see Deutsch and Silber 2005). Based on the same argumentation as for the component poverty line approach, I refrain from applying the former two as they do not allow for an attribute-wise consideration of poverty. The information theory approach has recently been extended to cover the component poverty line approach (Maasoumi and Lugo 2008). Its special appeal stems from the fact that it summarizes the information inherent in all attributes in an efficient manner. Nevertheless, the argumentation of this paper is that inequality is not only a concept of efficiency but also includes normative judgments. The axiomatic approach provides the most transparent way to take care of these judgments by explicitly defining properties that poverty indices may or may not satisfy.

Maasoumi and Lugo (2008, p.1) note that the axiomatic approach does well in aggregating across individuals but not across attributes. This paper follows their argumentation in suggesting that there might be good reason to deal with normative judgements on the individual level by utilising the axiomatic approach and with efficiency criteria by applying the information theory approach to ensure that attributes are aggregated in the most efficient manner. This way, both approaches can be combined, using the best of both of them.

This section starts with the introduction of a list of core axioms that have been derived by the generalization and extension of the core axioms of the one-dimensional framework to fit the multidimensional framework (e.g. Chakravarty, Mukherjee and Ranade 1998, Bourguignon and Chakravarty 1999, Tsui 2002, Bourguignon and Chakravarty 2003, Chakravarty and Silber 2008).

Non-Distributional Axioms

Anonymity (AN): For any $\mathbf{z} \in Z$ and $\mathbf{X} \in \mathcal{X}_n$, $P(\mathbf{X};\mathbf{z}) = P(\mathbf{\Pi X};\mathbf{z})$ where $\mathbf{\Pi}$ is any permutation matrix of appropriate order.

Continuity (CN): For any $\mathbf{z} \in Z$ and $\mathbf{X} \in \mathcal{X}_n$, $P(\mathbf{X};\mathbf{z})$ is continuous on $\mathbb{R}_+^{\mathbf{NK}}$.

Monotonicity (MN): For any $\mathbf{z} \in Z$ and $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_n$, if for any individual h and any attribute l $x_{hl} = x'_{hl} + \beta,$ such that $x'_{hl} < z_l, \beta > 0,$ and $x_{il} = x'_{il} \forall i \neq h,$ $x_{ij} = x'_{ij} \forall j \neq l, \forall i,$ then $P(\mathbf{X}';\mathbf{z}) \leq P(\mathbf{X};\mathbf{z}).$

Principle of Population (PP): If for any $\mathbf{z} \in Z$, $\mathbf{X} \in \mathcal{X}_n$, and $m \in \mathbb{N}$ $\mathbf{X}^m$ is a m-fold replication of $\mathbf{X}$, then $P(\mathbf{X}^m;\mathbf{z}) = P(\mathbf{X};\mathbf{z})$.

Strong Focus (SF): For any $\mathbf{z} \in Z$ and $\mathbf{X} \in \mathcal{X}_n$, if for any individual h and any attribute l $x_{hl} \geq z_l, x'_{hl} = x_{hl} + \beta, \beta > 0,$ and $x'_{il} = x_{il} \forall i \neq h$, $x'_{ij} = x_{ij} \forall j \neq l, \forall i,$ then $P(\mathbf{X};\mathbf{z}) = P(\mathbf{X}';\mathbf{z}).$

Subgroup Decomposability (SD): For any $\mathbf{X}^1,...,\mathbf{X}^v \in \mathcal{X}_n$ and $\mathbf{z} \in Z,$

$P(\mathbf{X}^1, \mathbf{X}^2,...,\mathbf{X}^v;\mathbf{z}) = \sum_{l=1}^{v} n_l / n P(\mathbf{X}^l;\mathbf{z})$ with n_l being the population size of subgroup $\mathbf{X}^l, l = 1,...,v$ and $\sum_{l=1}^{v} n_l = n.$

Factor Decomposability (FD): For any $\mathbf{z} \in Z$ and $\mathbf{X} \in \mathcal{X}_n$, $P(\mathbf{X};\mathbf{z}) = \sum_{j=1}^{k} a_j P(x_{.j}; z_j)$

Normalization (NM): For any $\mathbf{z} \in Z$ and $\mathbf{X} \in \mathcal{X}_n, P(\mathbf{X};\mathbf{z}) = 1$ if $x_{ij} = 0 \forall i, j$ and $P(\mathbf{X};\mathbf{z}) = 0$ if $x_{ij} \geq z_j \forall i, j.$ Thus, $P(\mathbf{X};\mathbf{z}) \in [0,1].$

AN requires that any personal characteristics apart from the respective achievement levels are irrelevant for poverty measurement. CN is a rather technical requirement precluding the oversensitivity of poverty measures. MN requires poverty measures not to increase if, ceteris paribus, the condition of a deprived individual improves. PP precludes the dependence of poverty measures from population size and thus allows for cross-population and -time comparisons of poverty. SF demands that giving a person more of an attribute with respect to which this person is not deprived will not change the poverty measure. FD and SD facilitate the calculation of the contribution of different subgroup-attribute combinations to overall poverty, improving the targeting of poverty-alleviating policies. NM is a simple technical property requiring poverty measures to be equal to zero in case all individuals are non-poor and equal to one in case all individuals are poor.

Distributional Axioms

I will now turn to the group of axioms that specifically deal with inequality issues. Scale Invariance (SI) requires that a *proportional* distribution should leave inequality levels unchanged, ensuring that poverty indices do not change with the unit of measurement.

Scale Invariance (SI): For any $\mathbf{z} \in Z$ and $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_\mathrm{n}$, $P(\mathbf{X};\mathbf{z}) = P(\mathbf{X}';\mathbf{z}')$ where $\mathbf{X}' = \mathbf{X\Lambda}$; $\mathbf{z}' = \mathbf{\Lambda z}$ with $\mathbf{\Lambda}$ being the diagonal matrix $diag(\lambda_1,...,\lambda_k), \lambda_j > 0 \forall j$.

In order to capture *inequality within dimensions*, poverty should not decrease in case the spread of dimension-specific achievements across society increases. In the one-dimensional context, this property is referred to as the *Pigou-Dalton Transfer Principle*. Different mathematical formulas have been used to extent the property to the multidimensional framework (de la Vega, Urrutia and de Sarachu, 2010). The one most widely used is the Uniform Majorization (UM) axiom.

Uniform Majorization (UM): For any $\mathbf{z} \in Z$ and $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_\mathrm{n}$, if $\mathbf{X}^P = \mathbf{B}\mathbf{X}'^P$ and $\mathbf{B}$ is not a permutation matrix, then $P(\mathbf{X};\mathbf{z}) \leq P(\mathbf{X}';\mathbf{z})$, where $\mathbf{X}^P\left(\mathbf{X}'^P\right)$ is the attribute matrix of the poor corresponding to $\mathbf{X}(\mathbf{X}')$ and $\mathbf{B} = \left(b_{ij}\right)$ is some bistochastic matrix of appropriate order.

UM requires that a transformation of the attribute matrix $\mathbf{X}'^P$ of the poor in $\mathbf{X}'$ into the corresponding matrix $\mathbf{X}^P$ of the poor in $\mathbf{X}$ by an equalising operation does not increase poverty.

As has been pointed out, in a multidimensional framework exists yet another aspect of inequality, namely *inequality between poverty dimensions*. This type of inequality has traditionally been equated with association sensitivity and captured by the concept of an *association increasing switch*.[5] The underlying majorization criterion has been proposed by Boland and Proschan (1988) and was generalized and formally introduced by Tsui (1999) as "Correlation Increasing Transfer".

Association Increasing Switch:[6] For any two vectors $\mathbf{x} = (x_1,..., x_k)$ and $\mathbf{x}' = (x'_1,..., x'_k)$ define the two operators $\barwedge$ and $\triangledown$ as follows: $\mathbf{x} \barwedge \mathbf{x}' = \left(\min\{x_1, x'_1\},...,\min\{x_k, x'_k\}\right)$ and $\mathbf{x} \triangledown \mathbf{x}' = \left(\max\{x_1, x'_1\},...,\max\{x_k, x'_k\}\right)$. For every $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_n$, $\mathbf{X}'$ is obtained from $\mathbf{X}$ by an association increasing switch if $\mathbf{X}'$ is not a permutation of $\mathbf{X}$ and if for some poor individuals *g* and *h*, $\mathbf{x}'_g = \mathbf{x}_g \barwedge \mathbf{x}_h$, $\mathbf{x}'_h = \mathbf{x}_g \triangledown \mathbf{x}_h$ and $\mathbf{x}'_m = \mathbf{x}_m \forall m \notin \{g, h\}$.

Consider two persons who – though both of them deprived in all attributes – face different achievement levels: each person has less than the other of at least one attribute. A switch of achievements is called association increasing if, after the switch, one of the two persons has at least as much as the other of *all* attributes.

For the purpose of illustration consider the following situation of three individuals and four attributes: $i = 3, j = 4, \mathbf{z} = (5 \quad 5 \quad 5 \quad 5)$ and $\mathbf{X} = \begin{bmatrix} 1 & 2 & 4 & 4 \\ 4 & 2 & 1 & 2 \\ 1 & 4 & 3 & 1 \end{bmatrix}$. Now, consider the following switches of achievements, first between individual one and individuals two and three, afterwards between individual two and three:

[5] Based on a paper of Chakravarty and D'Ambrosio (2006) on social exclusion measures, Jayaraj and Subramanian (2010) introduce inequality between dimensions as the spread of simultaneous deprivations across a society and based on this definition formulate the property "(Strong) Range Sensitivity". However, the authors fail to account for association-sensitivity which is why this paper refrains from employing these properties.

[6] Please note that the concept of the "Association Increasing Switch" is slightly different from the "Correlation Increasing Switch" formulated by Bourguignon and Chakravarty (2003). The latter definition is unclear as it

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 4 & 4 \\ 4 & 1 & 2 & 2 \\ 2 & 4 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 4 & 4 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Through the switches, individual one receives strictly higher, individual three strictly lower achievements in all attributes. Thus, the switches lead to a concentration of attributes and thus higher inequality. Based on the concept of association increasing switches, Tsui (1999) introduced the following property.

Nondecreasingness under Association Increasing Switch (NDA): For any $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_n$ such that $\mathbf{X}'$ is obtained from $\mathbf{X}$ by an association increasing switch of *substitute* attributes, $P(\mathbf{X};\mathbf{z}) \leq P(\mathbf{X}';\mathbf{z})$.

Bourguignon and Chakravarty (2003), however, claimed that in case attributes are complements, poverty should decrease even though association increasing switches lead to an increase in within dimensional inequality. In response, they introduce the following additional property.

Nonincreasingness under Association Increasing Switch (NIA): For any $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_n$ such that $\mathbf{X}'$ is obtained from $\mathbf{X}$ by an association increasing switch of *complement* attributes, $P(\mathbf{X};\mathbf{z}) \geq P(\mathbf{X}';\mathbf{z})$.

For the purpose of illustration consider left and right shoes to be the poverty attributes in question. Obviously, the two attributes are complements; a right shoe is only valuable in case it comes along with a left shoe to make it a pair. Let's assume an economy with two poor individuals and given poverty thresholds of 10 left and 10 right shoes per persons. Further, let one person have an initial endowment of 8 left and 2 right shoes and the other an initial endowment of 1 left and 3 right shoes: $i = 2, j = 2, \mathbf{z} = (10 \quad 10)$ and $\mathbf{X} = \begin{bmatrix} 8 & 2 \\ 1 & 3 \end{bmatrix}$. In other

requires an increase in the correlation between two attributes but leaves the correlation between all other attributes unaltered.

words, person one faces a surplus of 6 left shoes, person two a surplus of 2 right shoes. Obviously, the situation is not efficient. Indeed, two association increasing switches are possible that would enhance the overall situation. In the first one, the persons would exchange their amount of left shoes, i.e. $\mathbf{X}' = \begin{bmatrix} 1 & 2 \\ 8 & 3 \end{bmatrix}$, in the second their amount of right shoes, i.e.

$\mathbf{X}'' = \begin{bmatrix} 8 & 3 \\ 1 & 2 \end{bmatrix}$.

[Figure 2]

Both situations should be preferred to the initial one as an additional pair of shoes is made available. This is exactly the consideration behind NIA.

But there is an important difference between the two switches. Under the first switch, the second person gains two additional pairs of shoes, the first person, however, actually looses one pair. Under the second switch, the first person gains an additional pair of shoes whereas the situation of the second person remains unchanged. Though the overall outcome is the same, one person possessing one and the other three pairs of shoes, the processes that led to the respective outcomes are different. Whereas the first switch would be strongly opposed by the first person, the second switch would encounter much less opposition as it improves the situation of one person without worsening the situation of the other, a characteristic that has become known in economic theory as pareto-efficiency. For reasons that are obvious, pareto-efficiency is a rather valuable property for policy-makers. In the case of complements, pareto-efficiency can always be ensured if switches are restricted to cases in which the respective minimum achievement levels are not undercut. Thus, I extend the property NIA to ensure pareto-efficiency.

Nonincreasingness under Pareto-efficient Association Increasing Switch (NIPA): For any $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_n$ such that $\mathbf{X}'$ is obtained from $\mathbf{X}$ by an association increasing switch of *complement* attributes between two poor individuals *g* and *h* with $\min\{\mathbf{x}_g\} \leq \min\{\mathbf{x}_h\}$ and

$\mathbf{x}'_g = \mathbf{x}_g \barwedge \mathbf{x}_h$, $\mathbf{x}'_h = \mathbf{x}_g \triangledown \mathbf{x}_h$ and $\mathbf{x}'_m = \mathbf{x}_m \forall m \notin \{g, h\}$, then $P(\mathbf{X};\mathbf{z}) \geq P(\mathbf{X}';\mathbf{z})$.

In case all individuals are deprived in all dimensions, sensitivity to (pareto-efficient) association increasing switches in connection with UM accounts satisfactory for both inequality within and between dimensions.

But what if individuals suffer from different numbers of simultaneous deprivations? This is a more than legitimate question, especially since this case serves as the main justification for poverty measures that go beyond simple averages.

Consider the following situation: $i = 2, j = 5, \mathbf{z} = (5 \quad 5 \quad 5 \quad 5)$ and $\mathbf{X} = \begin{bmatrix} 1 & 2 & 5 & 5 \\ 2 & 1 & 5 & 1 \end{bmatrix}$ and

the following two possible switches: $\mathbf{X}' = \begin{bmatrix} 2 & 2 & 5 & 5 \\ 1 & 1 & 5 & 1 \end{bmatrix}$; $\mathbf{X}'' = \begin{bmatrix} 1 & 1 & 5 & 5 \\ 2 & 2 & 5 & 1 \end{bmatrix}$.

Both switches constitute a weaker version of the original association increasing switches as they are not limited to persons who are deprived in all attributes. Instead, switches among persons who are deprived in different numbers of attributes are allowed as long as the respective switches concern only attributes in which all persons affected by the switch are deprived. Thus, in the example above, the focus would be on the first two attributes.

This paper suggests that it is impossible to formulate any reasonable property that is based on a switch from $\mathbf{X}$ to either $\mathbf{X}'$ or $\mathbf{X}''$. The reason is that such a general property would be obliged to include in some way value judgments that weight the severity of inequality *within* against inequality *between* dimensions. As we will see later on, the new class of poverty indices derived in this paper captures this specific aspect with an interaction term.

A general assessment, however, can be made with regard to the question who – given the association increasing switch takes place – should be the beneficiary of the switch, i.e. should the switch to $\mathbf{X}'$ or $\mathbf{X}''$ be preferred? I suggest that the response to that question depends on the relationship between attributes. In case attributes are substitutes, the beneficiary of the switch should be the individual that is deprived in more attributes. In the example above, that would be $\mathbf{X}''$ as the beneficiary of the switch is the second individual that is deprived in three attributes instead of two. However, in case attributes are complements, pareto-efficient switches should be preferred, i.e. the individual with the higher minimum achievement level should be the beneficiary of the switch. In the example, that would be $\mathbf{X}'$ as the second individual has only one unit of the fourth attribute and therefore no use for any additional amount of attribute one or two. In response, I introduce the following concept of an extended version of the association increasing switch and, based on that definition, a new property called Inequality Sensitivity (IS).

Weak Association Increasing Switch: Define $d_i = \#\{c_{ij} \mid c_{ij} = 1\}$. For any two vectors

$\mathbf{x} = (x_1, ..., x_k)$ and $\mathbf{x}' = (x'_1, ..., x'_k)$ define the two operators $\overset{=}{\wedge}$ and $\overset{=}{\vee}$ as follows:

$\mathbf{x} \overset{=}{\wedge} \mathbf{x}' = (\min\{x_1, x'_1\}, ..., \min\{x_k, x'_k\} \forall x_j < z_j ; x_j = x'_j \forall x_j \geq z_j)$ and

$\mathbf{x} \overset{=}{\vee} \mathbf{x}' = (\max\{x_1, x'_1\}, ..., \max\{x_k, x'_k\} \forall x_j < z_j ; x_j = x'_j \forall x_j \geq z_j)$.

For every $\mathbf{X}, \mathbf{X}' \in \mathcal{X}_n$, $\mathbf{X}'$ is obtained from $\mathbf{X}$ by a *weak association increasing switch* if $\mathbf{X}'$ is not a permutation of $\mathbf{X}$ and if for some poor individuals *g* and *h*, $\mathbf{x}'_g = \mathbf{x}_g \overset{=}{\wedge} \mathbf{x}_h$, $\mathbf{x}'_h = \mathbf{x}_g \overset{=}{\vee} \mathbf{x}_h$ and $\mathbf{x}'_m = \mathbf{x}_m \forall m \notin \{g, h\}$.

Inequality Sensitivity (IS): Define $d_i = \#\{c_{ij} \mid c_{ij} = 1\}$. For some $\mathbf{X}, \mathbf{X}', \mathbf{X}'' \in \mathcal{X}_n$, if $\mathbf{X}'$ and $\mathbf{X}''$ are obtained from $\mathbf{X}$ by a *weak association increasing switch* between two poor individuals *g* and *h* with $d_g > d_h > 1$ such that

$\mathbf{x}'_g = \mathbf{x}_g \overset{=}{\wedge} \mathbf{x}_h$, $\mathbf{x}'_h = \mathbf{x}_g \overset{=}{\vee} \mathbf{x}_h$ and $\mathbf{x}'_m = \mathbf{x}_m$ for all $m \notin \{g, h\}$ and

$\mathbf{x}''_g = \mathbf{x}_g \overset{=}{\vee} \mathbf{x}_h$, $\mathbf{x}''_h = \mathbf{x}_g \overset{=}{\wedge} \mathbf{x}_h$ and $\mathbf{x}''_m = \mathbf{x}_m$ for all $m \notin \{g, h\}$,

then in case attributes are *substitutes* $P(\mathbf{X}''; \mathbf{z}) \leq P(\mathbf{X}'; \mathbf{z})$; in case attributes are *complements*, $P(\mathbf{X}''; \mathbf{z}) \leq P(\mathbf{X}'; \mathbf{z})$ if and only if $\min\{\mathbf{x}''_g\} \geq \min\{\mathbf{x}''_h\}$.

The concept of inequality increasing switches illustrates the previous observation that inequality between dimensions is closely related to the relationship between attributes yet not the same. The centre theme of the following section is the derivation and comparison of poverty indices satisfying different levels of sensitivity to inequality within and between dimensions.

Inequality-Sensitive Poverty Indices

Property 1. A multidimensional poverty measure P satisfies AN, CN, NM, MN, SF, PP, FD, SD, SI, UM and IS *if and only if* for all $n \in \mathbf{N}$ and $\mathbf{X} \in \mathcal{X}_n$:

$$P(\mathbf{X};\mathbf{z}) = 1/n \sum_{i \in S_j} h(\mathbf{c}_i) \sum_{j=1}^{k} a_j f(x_{ij} / z_j) \tag{1}$$

with $f:[0,\infty] \to R^1$ continuous, non-increasing and convex, with $f(0)=1$ and $f(t)=c$ for all $t \geq 1$ where $c<1$ is a constant. Also, $a_j > 0$ are constants with $\sum_{j=1}^{k} a_j = 1$.

Finally, $h: \mathbb{R}_{+}^{\mathbf{K}} \to [0,1]$ is nondecreasing with a nondecreasing (nonincreasing) marginal in case attributes are substitutes (complements).

The additive structure of the poverty measure is mandatory for the fulfilment of FD and automatically precludes sensitivity to association increasing switches. It also implies that sensitivity to inequality between dimensions can only be integrated in the final index through an adaptation in the identification step (Rippin 2012).

The aggregation of individual poverty characteristics into the overall index should comprise i) normative judgements in order to guide the allocation of scarce resources to the most needy, and ii) efficiency considerations in order to ensure that no scarce resources get wasted. The functional forms of $h(\cdot)$ and $f(\cdot)$ should ensure both.

$h(\mathbf{c}_i)$ is derived from the identification function $\rho_{CS}: \mathbb{R}_{+}^{\mathbf{K}} \times \mathbb{R}_{++}^{\mathbf{K}} \to [0,1]$ that differentiates individuals according to the severity with which they suffer deprivation, thereby ensuring that the neediest receive appropriate attention. In the following, I will concentrate on the following specific functional form of $h(\mathbf{c}_i)$ that has been chosen due to its appealing intuitive and simple design: $h(\mathbf{c}_i) = \begin{cases} \delta_i^{\alpha} & if \quad \max\{\mathbf{c}_i\} = 1 \\ 0 & if \quad \max\{\mathbf{c}_i\} = 0 \end{cases}$.

In other words, the degree of poverty severity is measured by the sum of weighted deprivations to the power α. The parameter α can be interpreted as an indicator for aversion towards inequality between dimensions, the value of which ought to depend on the relationship among attributes. In fact, choosing a value for α that is smaller than one directly implies the assumption that attributes are complements, enforcing a concave shape of $h(\mathbf{c}_i)$.

In this specific case, inequality between dimensions would actually be preferred, very much in the same sense as the intuition behind NIPA and IS. Choosing a value for α that is greater than one, on the other hand, directly determines a substitute relationship between attributes, enforcing a convex shape of $h(\mathbf{c}_i)$.

Several suggestions have been made with regard to the functional form of $f(\cdot)$. However, as already noted, it seems that the axiomatic approach with its normative judgements does well in aggregating across individuals but not across attributes. Thus, I utilise the following "optimal" IT aggregation functions to ensure that attributes are aggregated in an efficient manner, wasting no scarce resources (Maasoumi and Lugo, 2008):

$$S_i \propto \left[\sum_{j=1}^{k} w_j v_{ij}^{\delta}\right]^{1/\delta} \quad \text{when } \delta \neq 0 \tag{2}$$

$$S_i \propto \prod_{j=1}^{k} v_{ij}^{w_j} \quad \text{when } \delta = 0 \tag{3}$$

w_j being the weight attached to the Generalized Entropy divergence from each attribute.

Please note that the optimal IT aggregation function imposes a union definition of poverty in the sense that all information about the distribution of attributes is taken care of (Maasoumi and Lugo 2008, p. 10).

Utilising the component poverty approach, the following family of IT-efficient multi-dimensional poverty indices can be derived as the α th moment of the distribution $\mathbf{S}_v = (S_{v1},...,S_{vn})$:

$$P(\mathbf{S}_v;\mathbf{z}) = 1/n\sum_{i\in S_j} S_{vi}^{\gamma} \tag{4}$$

with S_{vi} representing the respective relative deprivation function according to (2) and (3).

In the following, I will introduce five of the most well-known (cardinal) classes of multidimensional poverty measures and discuss them under the aspect of IT-efficiency.

The first three classes have an additive structure and therefore lend themselves as functional forms for $f(\cdot)$ as specified in (1). The last two are non-additive.

The multidimensional Foster-Greer-Thorbecke (FGT) class of poverty measures

This class of poverty measures is a multidimensional extension of the Foster-Greer-Thorbecke index from 1984. The class is IT-efficient; the IT measure for $\delta \neq 0$ as specified in (2) is a version of this class of poverty measures in case $\gamma = \delta$ and $v_{ij} = 1 - x_{ij}/z_j$.

$$P_{FGT} = 1/n\sum_{i\in S_j} \sum_{j\in\{1,...,k\}:c_{ij}=1} a_j\left(1 - x_{ij}/z_j\right)^{\theta} \tag{5}$$

with $a_j > 0; \sum_{j=1}^{k} a_j = 1;\ \theta > 1$

Like α, θ can be interpreted as an indicator for aversion towards inequality within dimensions. However, different from α, θ is limited to values greater than one, reflecting the fact that it measures the aversion against inequality within every single dimension separately.

The first multidimensional Chakravarty class of poverty measures

This class of poverty measures is a direct multidimensional extension of the Chakravarty index from 1983. Like the previous class, this class of poverty measures is also IT efficient; the IT measure for $\delta \neq 0$ as specified in (2) is a version of this class of poverty measures in case $\gamma = \delta$ and $v_{ij}^{\delta} = 1 - (x_{ij}/z_j)^{\theta}$.

$$P_{C_1} = 1/n \sum_{i \in S_j} \sum_{j \in \{1,\dots,k\}: c_{ij}=1} a_j (1 - (x_{ij}/z_j)^{\theta}) \tag{6}$$

with $a_j > 0; \sum_{j=1}^{k} a_j = 1; \ \theta \in (0,1)$

This class of indices satisfies is comparable to the previous one, except for the fact that the progression of the function $f(\cdot)$ in this case is less regular in the sense that it is rather steep for very small values of x_{ij} and almost linear afterwards.

The multidimensional Watts class of poverty measures

This class of poverty measures is a direct multidimensional extension of the Watts index from 1968.

$$P_W = 1/n \sum_{i \in S_j} \sum_{j \in \{1,\dots,k\}: c_{ij}=1} a_j \log(z_j / x_{ij}) \tag{7}$$

with $a_j > 0; \sum_{j=1}^{k} a_j = 1$

A disadvantage of this class of poverty measures is that the degree of inequality aversion cannot be chosen, as it is simply the logarithm. Another disadvantage is that the measure is unbounded, i.e. its upper bound depends on the units chosen for the poverty thresholds z_j, and not defined for $x_{ij} = 0$. It is, however, IT-efficient; the IT measure for $\delta \neq 0$ as specified in (2) is a normalized version of this class of indices in case $\gamma = \delta$ and $v_{ij}^{\delta} = \log(z_j / x_{ij})$.

The second multidimensional Chakravarty class of poverty measures

This class of poverty measures is a non-additive multidimensional extension of the Chakravarty index from 1983, and has been introduced by Tsui (2002).

$$P_{C_2} = 1/n \sum_{i \in S_j} \left[\prod_{j \in \{1,\dots,k\}: c_{ij}=1} (z_j / x_{ij})^{r_j} - 1 \right] \tag{8}$$

with $r_j \in [0,1]$.

This class of poverty measures is no longer additive. Like the former class this class too is unbounded, i.e. its upper bound depends on the units chosen for z_j. Like the other indices, it is IT-efficient, precisely the IT measure for $\delta = 0$ (3) is a normalized version of this class of indices in case $\gamma = 1$ and $v_{ij}^{w_j} = (z_j / x_{ij})^{r_j} - 1$ (Maasoumi and Lugo, 2008, p. 9).

Bourguignon and Chakravarty (2003) criticised this class of poverty measures for restricting attributes to substitutes. In response, they introduced the following class of poverty measures:

The multidimensional Bourguignon-Chakravarty class of poverty measures

$$P_{BC} = 1/n \sum_{i \in S_j} \left[\sum_{j \in \{1,\ldots,k\}: c_{ij}=1} a_j \left(1 - x_{ij} / z_j\right)^{\theta} \right]^{\beta/\theta} \tag{9}$$

with $a_j > 0;\ \sum_{j=1}^{k} a_j = 1;\ \ \theta > 1; \beta \geq \theta \vee \beta \leq \theta$

As Chakravarty and Silber (2008) point out, this class of indices is less simple than Tsui's multidimensional extension since constant elasticity i) is defined between shortfalls rather than attributes, and ii) does not necessarily equal one. However, the most significant difference is that this class does not require attributes to be substitutes but instead allows them to be either substitutes $(\beta > \theta)$ or complements $(\beta < \theta)$. This class of indices resembles the class of IT measures for $\delta \neq 0$ with $v_{ij} = 1 - x_{ij} / z_j$, $\gamma = \beta$ and $\delta = \theta$.

Table 1 compares the different classes of poverty measures according to the properties that they do or do not satisfy. Please note that with the new identification method can be applied to all poverty measures. In case of the additive indices the new identification method ensures fulfilment of IS. All non-additive poverty measures satisfy IS anyway, however, in case of the multidimensional Bourguignon-Chakravarty class of poverty measures, no solution has been suggested so far that would also ensure the fulfilment of NIPA. Due to considerations with regard to the fulfilment of normalization, factor decomposability and the more regular progression of the function, the remainder of the paper will focus on the FGT functional form of $f(\cdot)$, defining the following class of poverty indices:

$$P_{IS} = 1/n \sum_{i \in S_j} \delta_i^{\alpha} \sum_{j \in \{1,\ldots,k\}: c_{ij}=1} a_j \left(1 - x_{ij} / z_j\right)^{\theta} \tag{10}$$

Table 1 provides on overview of which properties are satisfied by which indices.

[Table 1]

In order to analyse the effects of within and between dimensional inequality on poverty measurement, I will utilise the following representative of Alkire and Foster's M_0 class of indices as a base case: $M_0 = 1/n \sum_{i \in S_j} \sum_{\substack{j \in \{1,\dots,k\}: c_{ij}=1 \\ \wedge \delta_i \geq \delta_{IM}^{\min}}} a_j$.

To this index, I will compare the multidimensional extension of the FGT poverty index, i.e. $P_{FGT}(\mathbf{X};\mathbf{z}) = 1/n \sum_{i \in S_j} \sum_{j \in \{1,\dots,k\}: c_{ij}=1} a_j (1 - x_{ij}/z_j)^{\theta}$, and the new inequality sensitive poverty index, i.e. $P_{IS}(\mathbf{X};\mathbf{z}) = 1/n \sum_{i \in S_j} \delta_i^{\alpha} \sum_{j \in \{1,\dots,k\}: c_{ij}=1} a_j (1 - x_{ij}/z_j)^{\theta}$. However, before turning to the empirical application, I will decompose the two latter indices according to the three poverty components incidence, intensity and inequality[7].

The Decomposition of the Multidimensional FGT-Index

The following draws on a decomposition done by Aristondo, Lasso de la Vega and Urrutia (2010) for the one-dimensional case.

Proposition 2.

$P_{FGT}(\mathbf{X};\mathbf{z}) = H \sum_{j \in \{1,\dots,k\}: c_{ij}=1} a_j (q_j/q)\{[\mu_j(\mathbf{g})]^{\theta}[1 + (\theta(\theta-1))GE_{\theta}(\mathbf{g})]\}$, with

i) the headcount ratio, i.e. $H = (q/n)$, measuring the *incidence of poverty*,

ii) the aggregate poverty gap ratio for attribute *j*, i.e. $\mu_j(\mathbf{g}) = 1/q_j \sum_{i \in S_j} g_{ij}$, measuring the *intensity of poverty*, and

[7] Please note that due to its insensitivity with regard to any kind of inequality, M_0 can only be decomposed into the product of poverty incidence and intensity (Alkire and Santos 2010).

iii) the Generalized Entropy inequality index of the poverty gaps for attribute *j*, i.e. $GE_\theta(\mathbf{g}) = [1/(\theta(\theta-1))][1/q_j]\sum_{i\in S_j}\{[g_{ij}/\mu_j(\mathbf{g})]^\theta - 1\}$, capturing *within dimensional inequality*.

While the multidimensional FGT index does account for inequality within dimensions, it fails to do so for inequality between dimensions. This failure has been justified with the explanation that the index's (wanted) additivity prevents its sensitivity to association-increasing switches. However, as argued before, association-sensitivity influences inequality between dimensions yet is not the same. The implication of the more holistic approach taken in this paper becomes obvious once we consider the decomposition of the additive ISPI that comprises both components, within as well as between dimensional inequality.

The Decomposition of the Inequality Sensitive Poverty Index

Proposition 3.

$$P_{IS}(\mathbf{X};\mathbf{z}) = H \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j(q_j/q)[\mu(\boldsymbol{\delta})]^\alpha[\mu_j(\mathbf{g})]^\theta[1+(\alpha(\alpha-1))GE_\alpha(\boldsymbol{\delta})][1+(\theta(\theta-1))GE_\theta(\mathbf{g})][I(\mathbf{g},\boldsymbol{\delta})]$$

with

i) the headcount ratio, i.e. $H = (q/n)$, measuring the *incidence of poverty*,

ii) the aggregate deprivation count ratio, i.e. $\mu(\boldsymbol{\delta}) = 1/q\sum_{i\in S_j}\delta_i$, measuring the *intensity of poverty breadth*,

iii) the aggregate poverty gap ratio for attribute *j*, i.e. $\mu_j(\mathbf{g}) = 1/q_j\sum_{i\in S_j} g_{ij}$, measuring the *intensity of poverty depth* for attribute *j*,

iv) the GE inequality measure of deprivation counts, i.e. $GE_\alpha(\boldsymbol{\delta}) = [1/q(\alpha(\alpha-1))]\sum_{i\in S_j}[[\delta_i/\mu(\boldsymbol{\delta})]^\alpha - 1]$, measuring *inequality between dimensions,*

v) the GE inequality measure of poverty gaps for attribute *j*, i.e. $GE_\theta(\mathbf{g}) = [1/q_j(\theta(\theta-1))]\sum_{i\in S_j}[[g_{ij}/\mu_j(\mathbf{g})]^\theta - 1]$, measuring *within dimensional inequality* for attribute *j*, and, finally,

vi) an interaction term $I(\mathbf{g},\boldsymbol{\delta}) = [1/q_j\sum_{i\in S_j}\delta_i g_{ij}^\theta/\{[1/q\sum_{i\in S_j}\delta_i][1/q_j\sum_{i\in S_j}g_{ij}^\theta]\}]$, mapping the *interaction* between poverty gaps and deprivation counts.

The ISPI explicitly accounts for the fact that individuals may suffer from *multiple simultaneous deprivations*, a fact that is axiomatically captured by sensitivity to inequality and enables the most comprehensive decomposition of any additive index developed so far.

Empirical Application

This sub-section illustrates the implications of the new methodology developed in this paper with data from the Demographic and Health Survey (DHS). As the empirical application is based on a comparison with the inequality insensitive M_0 as base case it follows many of the choices of its most prominent representative, the Multidimensional Poverty Index (MPI) (Alkire and Santos 2010). Like the choice of the DHS data, nationally representative surveys that are mainly funded by the US Agency for International Development (USAID) and that Alkire and Santos (2010) privilege over other internationally comparable surveys. The final country sample consists of 28 countries for which more or less recent DHS surveys exist and that do not lack any of the indicators chosen for the poverty calculations.

In order to be able to apply cardinal poverty indices, a reasonably meaningful cardinal interpretation of attributes needs to be ensured. I am aware that this kind of choices is always problematic and disputable. However, as a discussion of better choices would go well beyond the scope of this theoretical paper, I will leave this to future research.

The following analysis will draw upon the following five equally weighted indicators: maternal health, child health, education, living conditions and asset endowment. A household is deprived in *maternal health* if any woman in reproductive age (15-49) has a BMI smaller than 18.5, and in *child health* if any child has a weight-for-age z-score below -2.5 according to WHO statistics. These two indicators differ from the rest of the indicators in the sense that they lack definite lower boundaries. Thus, appropriate boundaries are chosen on the basis of medical reports. In the case of the BMI, encyclopedia.com states that "*a BMI between 13 and 15 corresponds to 48 to 55 percent of desirable body weight for a given height and describes the lowest body weight that can sustain life*". In the case of weight-for-age z-scores, medical research of Bern et al. (1997) revealed that weight-for-age z-scores below -4.4 were no longer associated with an increased risk of mortality. In response, the minimum levels of 14 and -4.5 were chosen for the normalisation of BMI and z-scores, respectively. For all other indicators, the minimum level utilised for normalisation is the natural boundary zero.

A household is deprived in *education* if none of its members has at least five years of schooling.

In order to capture the *living conditions* of a household, I follow a methodology suggested by Bérenger and Bresson (2010) and derive a composite index that comprises quantitative and qualitative aspects of living conditions. Precisely, the number of sleeping rooms per head adjusted by household composition is utilised as an indicator for overcrowding that is refined through the application of a coefficient of penalty that addresses i) *structural quality* as indicated by flooring conditions and connection for power supply, and ii) the quality of *physical amenities* as indicated by the quality of drinking water, toilet facilities, and cooking fuel. For each of these equally weighted indicators, the threshold is the respective MDG standard as used for the calculation of the MPI. Following Bérenger and Bresson (2010), I choose 0.3 as threshold for the final composite index.

Finally, a weighted asset index captures household deprivation in *asset endowments*. It comprises the MPI items i) television (0.15), ii) bicycle (0.16), iii) radio (0.10), iv) telephone

(0.18), v) motorbike (0.21), and vi) refrigerator (0.20)[8]. According to the characteristics of the distribution, households with a weighted asset index below 0.27 that do not own a car or truck are considered deprived. Based on these indicators, M_0 is calculated with a dual cut-off of 20% of the weighted sum of indicators. The multidimensional FGT index and the Inequality Sensitive Poverty Index (ISPI) P_{IS} are calculated for the cases $\theta = \alpha = 1.5$ and $\theta = \alpha = 2$.

[Place table 1 here]

It is immediately obvious from table 1 that distinct rank changes are caused by utilising cardinal indices instead of the ordinal M_0. Sixteen countries experience rank changes once the multidimensional FGT index is applied instead of M_0, the highest change being a loss of seven places in the case of Liberia, which is actually huge given the relatively small sample size. As is obvious from the table, this change is mainly due to the high levels of poverty intensity within the two dimensions years of schooling and assets that only cardinal indices are able to capture. Interestingly, Liberia experiences yet another distinct rank change in case the ISPI is utilised instead of the FGT index. Intuitively, since poverty in Liberia is mainly concentrated in two dimensions, inequality between dimensions can be expected to be relatively low, reflected in a lower ISPI value. This is indeed the case. Liberia reduces a lot of the losses induced by its within-dimensional failures in the dimensions education and assets and gains five places back in the ranking once the ISPI is utilised instead of the FGT index. India, on the other hand, has a rather low degree of inequality within dimensions so that it gains four places in the ranking once the FGT index is utilised in place of M_0. However, poverty intensity and inequality between dimensions, though not high, are nevertheless distinct, reducing the places gained to two once the ISPI is utilised in place of the FGT index.

[8] Brackets contain the weights of the respective items, calculated as the inverse of the frequency with which these items are observed across the sample.

Yet another interesting case is Nigeria. Nigeria demonstrates a combination of slightly increased within and between dimensional inequality when compared to its reference countries in the ranking. This characteristic induces a loss of two places once the FGT index is applied instead of M_0 and a loss of yet another two places once the ISPI is applied instead of the FGT index.

These examples plainly illustrate that the characteristics of poverty in a specific country are more and more uncovered through the change from M_0 to the FGT index to the ISPI. The importance that is attributed to these characteristics depends of course on the individual choices of θ and α, the parameters that express the aversion against within and between dimensional inequality.

[Place table 2 here]

Table 2 summarizes the results for the case that parameter values are increased from $\theta = \alpha = 1.5$ to $\theta = \alpha = 2$, indicating increased levels of inequality aversion. The resulting changes affect especially those countries that either show rather low or rather high levels of inequality, as the significance of outliers gets more pronounced as the level of inequality-aversion increases. Nigeria, for instances, looses two additional places in the ranking, one place is lost through the change from M_0 to the FGT index, the other through the change from the FGT index to the ISPI.

The empirical results reveal the importance of accounting for within and between dimensional inequality: The character of poverty is very different from country to country and the more comprehensively a poverty measure accounts for this, the more accurate is the insight gained into the very character of poverty in a region, country, district etc. This additional insight bears the potential to increase precision and effectiveness of poverty reducing strategies.

Conclusion

Inequality between dimensions is usually treated as association-sensitivity. However, such an equation seems to be too narrow and has some serious implications on the axiomatic foundation of multidimensional poverty indices. The definition of association-increasing switches as defined so far concentrates solely on the effects of association increases in dependence of the kind of attributes that are involved, i.e. whether the attributes that are switched are substitutes or complements. It neglects the issue of who the beneficiary of the respective switch is and how poverty indices might or might not change with a switch of beneficiaries.

In fact, in case the respective attributes are complements, association-increasing switches as they are defined today violate the economic principle of pareto-efficiency. This paper introduces an additional axiom that ensures pareto-efficiency of association-increasing switches.

But the issue goes even further; in fact it comprises the broader question what happens in case of switches between individuals that are deprived in a different number of dimensions. It is a highly relevant question that is a direct consequence of the restrictive interpretation of inequality between dimensions and in fact reveals that inequality is more than association-sensitivity. More precisely, this paper follows a definition already introduced by the author in a previous paper (2012), defining inequality between dimensions as the association-sensitive spread of simultaneous deprivations across a society. In consequence, this paper suggests the introduction of a switch between individuals that are deprived in a different number of dimensions whose effect on poverty does not only depend on the relationship among attributes but also on the choice of the beneficiary of the respective switch. The paper demonstrates how the new axiom can be utilised to derive a whole new class of poverty indices. This class is unique in the sense that it is the first class of additive poverty indices that i) explicitly accounts for inequality between dimensions as the association-sensitive spread of simultaneous deprivations across society, and, as a result, ii) improves the precision and detailedness of poverty profiles, thereby enhancing the targeting of poverty reduction policies.

Though this paper constitutes only a first step towards the measurement of inequality between dimensions in a broader sense, the empirical application in this paper plainly reveals its relevance and the need for further research in this important area.

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APPENDIX

Proof of Proposition 1.

The 'if' part of the proposition is straightforward to verify. To prove the 'only if' part, I proceed by induction on population size (see also Rippin 2012). Suppose that the new index $P(\mathbf{X};\mathbf{z})$ satisfies the axioms stated in the proposition.

Individual *i* is deprived in attribute *j* if $x_{ij} < z_j$, i.e. $c_{ij} = 1$. Likewise, $c_{ij} = 0$ if $x_{ij} \geq z_j$.

Now suppose $\mathbf{X} \in \mathcal{X}_1$. Let $\underline{\mathbf{x}}_1$ denote a vector of achievements with $\underline{x}_{1j} < z_j$ for all *j* and $\underline{\underline{\mathbf{x}_1}}$ a vector with zero achievement in all attributes, i.e. $\underline{\underline{x_{1j}}} = 0$ for all *j*. Finally, let $\bar{\mathbf{x}}_1$ be a vector of achievements with $\bar{x}_{1j} \geq z_j$ for all *j*. Then by normalization (NM), $P(\underline{\underline{\mathbf{x}_1}}) = 1$ and $P(\bar{\mathbf{x}}_1) = 0$. Let $f(\mathbf{c}_{1.}) \in [0,1]$ denote the general identification function of the poor. From monotonicity (MN) and inequality sensitivity (IS) it follows that $f(\mathbf{c}_{1.})$ is increasing in $\mathbf{c}_{1.}$ with a nondecreasing (nonincreasing) marginal in case attributes are substitutes (complements). Thus, $\max\{f(\mathbf{c}_{1.})\} = 1$ for all $\mathbf{x}_{1.} \in \underline{\mathbf{x}_1}$, expressing absolute poverty and $\min\{f(\mathbf{c}_{1.})\} = 0$ for all $\mathbf{x}_{1.} \in \overline{\mathbf{x}_1}$, identifying the case of no poverty.

Suppose $\mathbf{X} \in \mathcal{X}_1 \backslash \{\bar{\mathbf{x}}_1\}$. Then there exists at least one achievement level $\tilde{x}_{1j} \in \mathbf{X}$ with $\tilde{x}_{ij} \leq z_j$ for some $j \in \{1,...,k\}$. Then, $P(\tilde{x}_{1j}) = f(\mathbf{c}_{1.}) a_j g(\tilde{x}_{1j}; z_j)$.

Aggregating under factor decomposability (FD) leads to the general formula

$$P(\mathbf{X};\mathbf{z}) = \sum_{j \in \{1,...,k\}: c_{1j}=1} f(\mathbf{c}_{1.}) a_j g(x_{1j}; z_j) = f(\mathbf{c}_{1.}) \sum_{j \in \{1,...,k\}: c_{1j}=1} a_j \, g(x_{1j}; z_j). \tag{1}$$

where $a_j > 0$ and $\sum_{j=1}^{k} a_j = 1$. Due to scale invariance (SI), $g(x_{ij}; z_j) = g(x_{ij} / z_j)$ for all $(\mathbf{X};\mathbf{z}) \in \mathbf{K} \times \mathbf{Z}$ so that I can rewrite (1) as

$$P(\mathbf{X};\mathbf{z}) = f(\mathbf{c}_{1.}) \sum_{j \in \{1,...,k\}: c_{1j}=1} a_j \, g(x_{1j} / z_j) \tag{2}$$

with $g[0,\infty] \to R^1$ being continuous and non-increasing due to continuity (CN) and monotinicity (MN). Also, fulfilment of uniform majorization (UM) requires convexity of $g(.)$ (see Chakravarty, Mukherjee and Ranade 1998, p. 184). Finally, due to normalization, $P(\underline{\underline{\mathbf{x}_1}}) = \sum_{j=1}^{k} a_j g(0) = g(0)\sum_{j=1}^{k} a_j = g(0) = 1$. In addition, strong focus (SF) implies that $g(t) = c$ for all $t \geq 1$ with $c < 1$ being a constant. Please note that $P(\overline{\mathbf{x}_1}) = 0$ as required by normalization (NM) is already satisfied by $\min\{f(\mathbf{c}_{1.})\} = 0$ for all $\mathbf{x}_{1.} \in \overline{\mathbf{x}_1}$.

Suppose proposition 1 is true for all $n \in \mathbf{N}$.

Now, let $\mathbf{X} \in \mathcal{X}_{n+1}$, $\mathbf{X}' = \{\mathbf{x}_{ij} \big| i \in \{1,...,n\}, j \in \{1,...,k\}\}$ and $\mathbf{X}'' = \{\mathbf{x}_{ij} \big| i = n+1, j \in \{1,...,k\}\}$. (3)

When extending $f(\mathbf{c}_{1.})$ to a society with *n* individuals, the identification function in its most general form may *i)* depend on the deprivation vectors of other individuals, *ii)* differ across individuals, *iii)* depend on the population size *n*.

The first possibility is immediately ruled out by subgroup decomposability (SD), i.e. $f_i^n(\mathbf{c}_{i.} \times \{\mathbf{c}_{1.},...,\mathbf{c}_{i-1.},\mathbf{c}_{i+1.},...,\mathbf{c}_{n.},\mathbf{c}_{n+1.}\}) = f_i^n(\mathbf{c}_{i.})$ for all $i \in \mathbf{N}$. With this, I can rewrite (3) as

$$P(\mathbf{X};\mathbf{z}) = \frac{n}{n+1}P(\mathbf{X}';\mathbf{z}) + \frac{1}{n+1}P(\mathbf{X}'';\mathbf{z}) \Leftrightarrow$$

$$P(\mathbf{X};\mathbf{z}) = \left(\frac{n}{n+1}\right)\left(\frac{1}{n}\right)\sum_{i=1}^{n} f_i^{n+1}(\mathbf{c}_{i.}) \sum_{j\in\{1,...,k\}:c_{ij}=1} a_j g(x_{ij}/z_j) + \left(\frac{1}{n+1}\right) f_{n+1}^{n+1}(\mathbf{c}_{n+1.}) \sum_{j\in\{1,...,k\}:c_{n+1j}=1} a_j\, g(x_{ij}/z_j) \quad (4)$$

Next, I will show that the second possibility can be excluded, i.e. $f_i^n = f_{i'}^n$ for all $i,i' \in \mathbf{N}$.

Consider any $\hat{i},\tilde{i} \in \mathbf{N}$. Let $\mathbf{X} \in \mathcal{X}_n$ whereby $\mathbf{x}_{\hat{i}.} = \hat{\mathbf{x}}$ with $\underline{\mathbf{x}} \neq \hat{\mathbf{x}} \neq \overline{\mathbf{x}}$ and $\mathbf{x}_{i.} = \overline{\mathbf{x}}$ for all $i \neq \hat{i}$.

Likewise, let $\mathbf{X}' \in \mathcal{X}_n$ be such that $\mathbf{x}'_{\tilde{i}.} = \hat{\mathbf{x}}$ and $\mathbf{x}'_{i.} = \overline{\mathbf{x}}$ for all $i \neq \tilde{i}$. Using normalization (NM) and subgroup decomposition (SD):

$P(\mathbf{X};\mathbf{z}) = (n-1)/n + f_{\hat{i}}^n(\hat{\mathbf{c}}_.) \sum_{j\in\{1,...,k\}:\hat{c}_j=1} a_j\, g(\hat{x}_{.j}/z_j)$ and

$P(\mathbf{X}';\mathbf{z}) = (n-1)/n + f_{\tilde{i}}^n(\hat{\mathbf{c}}_.) \sum_{j\in\{1,...,k\}:\hat{c}_j=1} a_j\, g(\hat{x}_{.j}/z_j)$. From anonymity (AN) it follows that

$P(\mathbf{X};\mathbf{z}) = P(\mathbf{X}';\mathbf{z})$ and thus $f_{\hat{i}}^{n}(\hat{\mathbf{c}}_{.}) = f_{\tilde{i}}^{n}(\hat{\mathbf{c}}_{.})$. Hence, $f_i^n = f_{i'}^n$ for all $i, i' \in \mathbf{N}$. I denote this common function f^n.

Finally, also the third possibility can be excluded, i.e. $f^n = f^{n'}$ for all $n, n' \in \mathbf{N}$.

Consider any $\mathbf{X} \in \mathcal{X}_1$ so that $\mathbf{x}_{1.} = \hat{\mathbf{x}}_{.}$ is any achievements vector in **X**. Thus,

$P(\mathbf{X};\mathbf{z}) = f^1(\hat{\mathbf{c}}_{.}) \sum_{j\in\{1,\dots,k\}:\hat{c}_j=1} a_j\, g(\hat{x}_{.j}/z_j)$. Now, consider any $\hat{\mathbf{X}} \in \mathcal{X}_n$ so that $\hat{\mathbf{X}} = [\mathbf{X}]_n$ and

$\mathbf{z} \in \mathbf{Z} = \hat{\mathbf{z}} \in \mathbf{Z}$. Then, by population principle (PP) $P(\mathbf{X};\mathbf{z}) = P(\hat{\mathbf{X}};\mathbf{z})$, i.e.

$$P(\hat{\mathbf{X}};\mathbf{z}) = 1/n \sum_{i=1}^{n} f^n(\hat{\mathbf{c}}_{.}) \sum_{j\in\{1,\dots,k\}:\hat{c}_j=1} a_j g(\hat{x}_{.j}/z_j) = f^n(\hat{\mathbf{c}}_{.}) \sum_{j\in\{1,\dots,k\}:\hat{c}_j=1} a_j g(\hat{x}_{.j}/z_j) \overset{!}{=}$$

$$f^1(\hat{\mathbf{c}}_{.}) \sum_{j\in\{1,\dots,k\}:\hat{c}_j=1} a_j\, g(\hat{x}_{.j}/z_j) = P(\mathbf{X};\mathbf{z})$$

As a result, $f^1(\hat{\mathbf{c}}_{.}) = f^n(\hat{\mathbf{c}}_{.})$ and thus $f^{n'} = f^n$ for all $n, n' \in \mathbf{N}$. I denote this common function f.

With this I can rewrite equation (4) as

$$P(\mathbf{X};\mathbf{z}) = \left(\frac{n}{n+1}\right)\left(\frac{1}{n}\right)\sum_{i=1}^{n} f(\mathbf{c}_{i\cdot}) \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j\, g(x_{ij}/z_j) + \left(\frac{1}{n+1}\right) f(\mathbf{c}_{n+1\cdot}) \sum_{j\in\{1,\dots,k\}:c_{n+1j}=1} a_j\; g(x_{n+1j}/z_j) \Leftrightarrow$$

$$P(\mathbf{X};\mathbf{z}) = \frac{1}{n+1}\sum_{i=1}^{n+1} f(\mathbf{c}_{i\cdot}) \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j\, g(x_{ij}/z_j)$$

Q.E.D.

Proof of Proposition 2.

$$P_{FGT}(\mathbf{X};\mathbf{z}) = 1/n \sum_{i\in S_j} \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j g_{ij}^{\theta}$$

$$= 1/n \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j \sum_{i\in S_j} \left[g_{ij}\left((z_j - \mu_j(\mathbf{g}))/z_j\right)\left(z_j/(z_j - \mu_j(\mathbf{g}))\right)\right]^{\theta}$$

$$= 1/n \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j \left[(z_j - \mu_j(\mathbf{g}))/z_j\right]^{\theta} \sum_{i\in S_j} \left[g_{ij}\left(z_j/(z_j - \mu_j(\mathbf{g}))\right)\right]^{\theta}$$

$$= 1/n \sum_{j\in\{1,\dots,k\}:c_{ij}=1} a_j \left[1/z_j\left(1/q_j \sum_{i\in S_j} z_j - 1/q_j \sum_{i\in S_j} x_{ij}\right)\right]^{\theta}.$$

$$\sum_{i\in S_j}\left[g_{ij}\Big/\left(1/z_j\left[1/q_j\sum\nolimits_{i\in S_j} z_j - 1/q_j\sum\nolimits_{i\in S_j} x_{ij}\right]\right)\right]^{\theta}$$

$$= 1/n\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\left[1/q_j\sum\nolimits_{i\in S_j} g_{ij}\right]^{\theta}\sum_{i\in S_j}\left[g_{ij}\Big/\left(1/q_j\sum\nolimits_{i\in S_j} g_{ij}\right)\right]^{\theta}$$

$$= q/n\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j(1/q)\left[\mu_j(\mathbf{g})\right]^{\theta} q_j\left[1+(\theta^2-\theta)\left[\left(1/q_j(\theta^2-\theta)\right)\sum\nolimits_{i\in S_j}\left[g_{ij}/\mu_j(\mathbf{g})\right]^{\theta}-1\right]\right]$$

$$= H\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\left(q_j/q\right)\left[\mu_j(\mathbf{g})\right]^{\theta}\left[1+\left(\theta^2-\theta\right)GE_{\theta}(\mathbf{g})\right]$$

Q.E.D.

Proof of Proposition 3.

$$P_{IS}(\mathbf{X};\mathbf{z}) = 1/n\sum\nolimits_{i\in S_j}\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\,\delta_i^{\alpha} g_{ij}^{\theta}$$

$$= 1/n\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\sum\nolimits_{i\in S_j}\delta_i^{\alpha}\left[\sum\nolimits_{i\in S_j}\left(\delta_i/\mu(\boldsymbol{\delta})\right)^{\alpha}\right]1/\sum\nolimits_{i\in S_j}\left(\delta_i/\mu(\boldsymbol{\delta})\right)^{\alpha}\Big]\cdot$$

$$g_{ij}^{\theta}\left[\sum\nolimits_{i\in S_j}\left(g_{ij}/\mu_j(\mathbf{g})\right)^{\theta}\right]1/\sum\nolimits_{i\in S_j}\left(g_{ij}/\mu_j(\mathbf{g})\right)^{\theta}\Big]$$

$$= q/n\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\,(q_j/q)\sum\nolimits_{i\in S_j}(1/q_j)\delta_i^{\alpha} g_{ij}^{\theta}\left[1/\sum\nolimits_{i\in S_j}\left(\delta_i/\mu(\boldsymbol{\delta})\right)^{\alpha}\right]1/\sum\nolimits_{i\in S_j}\left(g_{ij}/\mu_j(\mathbf{g})\right)^{\theta}\Big]\cdot$$

$$\left[\sum\nolimits_{i\in S_j}\left(g_{ij}/\mu_j(\mathbf{g})\right)^{\theta}\right]\sum\nolimits_{i\in S_j}\left(\delta_i/\mu(\boldsymbol{\delta})\right)^{\alpha}\Big]$$

$$= q/n\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\,\left(q_j/q\right)\left[\mu(\boldsymbol{\delta})\right]^{\alpha}\left[\mu_j(\mathbf{g})\right]^{\theta}\left(1/q_j\sum\nolimits_{i\in S_j}\delta_i^{\alpha} g_{ij}^{\theta}\Big/\left[\left(1/q\sum\nolimits_{i\in S_j}\delta_i^{\alpha}\right)\left(1/q_j\sum\nolimits_{i\in S_j} g_{ij}^{\theta}\right)\right]\right)\cdot$$

$$\left[1+\left(\theta^2-\theta\right)/q_j\left(\theta^2-\theta\right)\right]\left[\sum\nolimits_{i\in S_j}\left[g_{ij}/\mu_j(\mathbf{g})\right]^{\theta}-1\right]\cdot$$

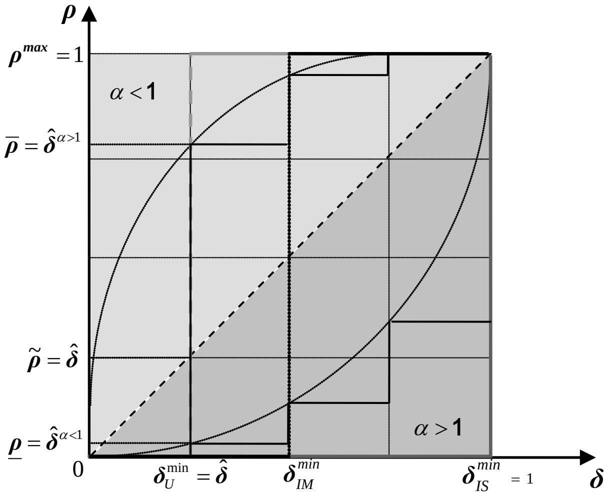
$$\left[1+\left(\alpha^2-\alpha\right)/q\left(\alpha^2-\alpha\right)\right]\left[\sum\nolimits_{i\in S_j}\left[\delta_i/\mu(\boldsymbol{\delta})\right]^{\alpha}-1\right]$$

$$= H\sum_{j\in\{1,\ldots,k\}:c_{ij}=1} a_j\left(q_j/q\right)\left[\mu(\boldsymbol{\delta})\right]^{\alpha}\left[\mu_j(\mathbf{g})\right]^{\theta} I(\mathbf{g},\boldsymbol{\delta})\left(1+\left(\theta^2-\theta\right)GE_{\theta}(\mathbf{g})\right)\left(1+\left(\alpha^2-\alpha\right)GE_{\alpha}(\boldsymbol{\delta})\right)$$

Q.E.D.

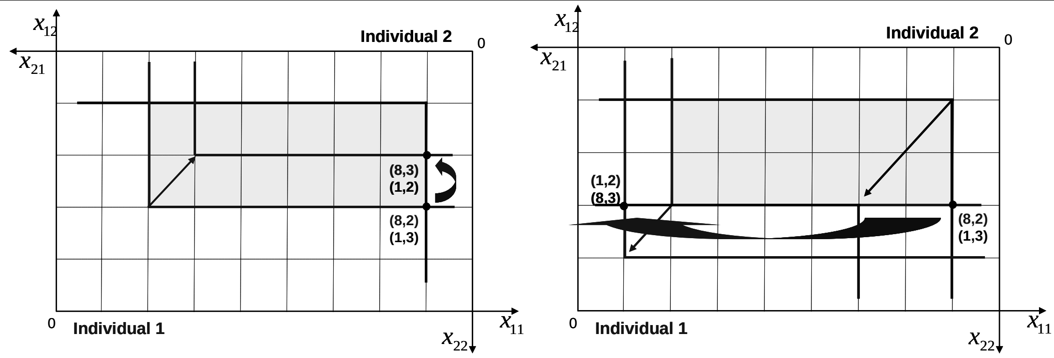
APPENDIX B

Fig. 1 The Correlation Sensitive Identification Method



Source: Own compilation

Fig 2 Pareto-Efficiency and Association Increasing Switches



Source: Own compilation

Tab 1 The Axiomatic Foundation of Selected Classes of Cardinal Poverty Measures

Axioms	P_{FGT}	P_{C1}	P_W	P_{C2}	P_{BC}	P_{IS}
Anonymity (AN)	✓	✓	✓	✓	✓	✓
Continuity (CN)	✓	✓	✓	✓	✓	✓
Monotonicity (MN)	✓	✓	✓	✓	✓	✓
Principle of Population (PP)	✓	✓	✓	✓	✓	✓
Strong Focus (SF)	✓	✓	✓	✓	✓	✓
Subgroup Decomposability (SD)	✓	✓	✓	✓	✓	✓
Factor Decomposability (FD)	✓	✓	✓	✗	✗	✓
Normalization (NM)	✓	✓	(✓)	✓	✓	✓
Scale Invariance (SI)	✓	✓	✓	✓	✓	✓
Uniform Majorization (UM)	✓	✓	✓	✓	✓/✗[15]	✓
Nondecreasingness under Association Increasing Switch (NDA)	✗	✗	✗	✓	✓/✗[15]	✗
Nonincreasingness under Association Increasing Switch (NIA)	✗	✗	✗	✗	✓/✗[16]	✗
Nonincreasingness under Pareto-efficient Association Increasing Switch (NIPA)	✗	✗	✗	✗	✗	✗
Inequality Sensitivity (IS)	✗	✗	✗	✓	✓	✓

[15] Only satisfied in case attributes are substitutes, i.e. for $\delta > \alpha$

[16] Only satisfied in case attributes are complements, i.e. for $\delta < \alpha$

Tab. 2: Decomposition of FGT and ISPI, α = 1.5 (alphabetical ordering)

				σ					μ(g)					GE(g)							I(g,d)				
Country	Δ	FGT	H	g_1	g_2	g_3	g_4	g_5	g_1	g_2	g_3	g_4	g_5	g_1	g_2	g_3	g_4	g_5	μ(d)	GE(d)	g_1	g_2	g_3	g_4	g_5
Armenia	+1	0.009	0.246	0.24	0.05	0.02	0.46	0.37	0.245	0.298	0.444	0.253	0.491	0.352	0.378	0.205	0.229	0.104	0.203	0.145	0.960	1.005	2.109	1.637	0.686
Azerbaijan	-	0.016	0.360	0.17	0.10	0.02	0.66	0.33	0.248	0.327	0.538	0.262	0.440	0.340	0.331	0.182	0.174	0.160	0.250	0.107	1.157	1.436	1.896	1.374	0.888
Bangladesh	+4	0.117	0.829	0.35	0.27	0.30	0.63	0.79	0.310	0.319	0.574	0.350	0.748	0.270	0.311	0.145	0.142	0.099	0.424	0.178	1.369	1.655	1.857	1.469	1.220
Benin	-2	0.147	0.841	0.13	0.24	0.52	0.66	0.60	0.246	0.359	0.687	0.335	0.459	0.342	0.356	0.112	0.158	0.355	0.452	0.121	1.462	1.562	1.424	1.255	1.078
Bolivia	-1	0.063	0.663	0.03	0.06	0.16	0.77	0.70	0.188	0.325	0.472	0.398	0.404	0.479	0.397	0.165	0.133	0.396	0.322	0.128	1.033	2.016	1.959	1.245	1.295
Cambodia	+1	0.140	0.927	0.24	0.15	0.28	0.92	0.44	0.260	0.302	0.462	0.508	0.483	0.321	0.332	0.169	0.070	0.286	0.436	0.097	1.424	1.924	1.765	1.053	1.396
Cameroon	-	0.097	0.785	0.10	0.16	0.32	0.47	0.85	0.306	0.355	0.661	0.301	0.626	0.310	0.311	0.118	0.175	0.166	0.328	0.262	1.901	2.159	1.946	1.685	0.998
Congo, Rep.	-	0.075	0.824	0.21	0.14	0.08	0.67	0.79	0.303	0.329	0.538	0.319	0.717	0.299	0.343	0.162	0.166	0.100	0.314	0.163	1.505	2.011	2.098	1.416	1.127
DR Congo	-	0.114	0.916	0.22	0.24	0.17	0.70	0.84	0.279	0.412	0.563	0.371	0.679	0.325	0.310	0.153	0.136	0.162	0.374	0.151	1.557	1.706	1.835	1.311	1.105
Ethiopia	-	0.305	0.982	0.24	0.25	0.63	0.91	0.98	0.289	0.409	0.724	0.537	0.869	0.342	0.310	0.095	0.069	0.030	0.602	0.068	1.279	1.442	1.262	1.087	1.056
Ghana	-3	0.076	0.711	0.06	0.06	0.23	0.70	0.60	0.241	0.306	0.666	0.335	0.543	0.300	0.433	0.115	0.158	0.231	0.325	0.144	1.671	1.882	1.882	1.210	1.095
Haiti	-3	0.146	0.883	0.19	0.11	0.40	0.67	0.85	0.317	0.428	0.635	0.385	0.683	0.329	0.320	0.116	0.132	0.141	0.419	0.151	1.616	1.850	1.572	1.386	1.200
India	+4	0.132	0.846	0.47	0.26	0.22	0.74	0.68	0.356	0.423	0.679	0.407	0.582	0.244	0.283	0.122	0.121	0.186	0.440	0.135	1.264	1.737	1.910	1.302	1.247
Kenya	-	0.102	0.887	0.17	0.14	0.14	0.71	0.91	0.276	0.344	0.635	0.399	0.529	0.322	0.373	0.144	0.133	0.260	0.347	0.183	1.874	2.097	2.368	1.452	1.213
Liberia	-7	0.150	0.904	0.14	0.18	0.33	0.69	0.79	0.238	0.377	0.734	0.370	0.807	0.335	0.335	0.098	0.133	0.046	0.398	0.144	1.498	1.518	1.711	1.244	1.135
Malawi	+1	0.120	0.951	0.09	0.16	0.30	0.69	0.97	0.257	0.344	0.528	0.384	0.498	0.374	0.398	0.139	0.131	0.334	0.389	0.173	1.698	1.762	1.840	1.368	1.123
Mali	-	0.228	0.909	0.19	0.32	0.68	0.67	0.62	0.279	0.434	0.797	0.357	0.461	0.327	0.291	0.076	0.144	0.336	0.531	0.093	1.321	1.402	1.242	1.261	1.161
Moldova	-1	0.009	0.228	0.21	0.03	0.16	0.20	0.59	0.234	0.175	0.391	0.240	0.466	0.342	0.486	0.259	0.238	0.303	0.181	0.246	1.002	0.920	2.491	1.996	0.992
Morocco	+2	0.070	0.578	0.20	0.10	0.45	0.55	0.50	0.250	0.369	0.578	0.278	0.528	0.298	0.366	0.164	0.202	0.198	0.381	0.139	1.067	1.763	1.502	1.424	1.312
Mozambique	-	0.161	0.938	0.10	0.19	0.60	0.61	0.96	0.205	0.382	0.573	0.334	0.555	0.388	0.375	0.146	0.157	0.269	0.472	0.141	1.403	1.666	1.369	1.422	1.075
Namibia	+3	0.059	0.632	0.33	0.18	0.14	0.53	0.60	0.314	0.309	0.584	0.334	0.670	0.313	0.426	0.153	0.189	0.104	0.310	0.188	1.330	2.022	2.243	1.599	1.196
Nepal	+4	0.138	0.903	0.34	0.29	0.34	0.64	0.86	0.309	0.343	0.629	0.375	0.558	0.278	0.300	0.129	0.142	0.203	0.443	0.178	1.455	1.781	1.744	1.448	1.161
Niger	-	0.296	0.971	0.21	0.40	0.68	0.77	0.90	0.271	0.460	0.844	0.387	0.739	0.287	0.254	0.054	0.131	0.077	0.595	0.079	1.364	1.324	1.220	1.173	1.054

Tab. 3: Decomposition of FGT and ISPI, $\alpha = 2$ (alphabetical ordering)

				$\sigma = q_j/q$					$\mu(g)$					$GE(g)$							$I(g,d)$					
Country	Δ	FGT	H	g_1	g_2	g_3	g_4	g_5	g_1	g_2	g_3	g_4	g_5	g_1	g_2	g_3	g_4	g_5	$\mu(d)$	$GE(d)$	g_1	g_2	g_3	g_4	g_5	ISPI
Armenia	+1	0.006	0.246	0.24	0.05	0.02	0.46	0.37	0.245	0.298	0.444	0.253	0.491	0.373	0.423	0.217	0.247	0.107	0.203	0.140	0.941	0.898	2.442	1.761	0.737	0.000
Azerbaijan	-	0.011	0.360	0.17	0.10	0.02	0.66	0.33	0.248	0.327	0.538	0.262	0.440	0.369	0.353	0.181	0.179	0.162	0.250	0.102	1.337	1.698	2.199	1.475	0.916	0.001
Bangladesh	+4	0.096	0.829	0.35	0.27	0.30	0.63	0.79	0.310	0.319	0.574	0.350	0.748	0.278	0.330	0.143	0.138	0.090	0.424	0.176	1.455	1.879	2.061	1.590	1.297	0.040
Benin	-2	0.126	0.841	0.13	0.24	0.52	0.66	0.60	0.246	0.359	0.687	0.335	0.459	0.368	0.378	0.106	0.156	0.338	0.452	0.118	1.632	1.775	1.515	1.318	1.107	0.047
Bolivia	-1	0.048	0.663	0.03	0.06	0.16	0.77	0.70	0.188	0.325	0.472	0.398	0.404	0.573	0.431	0.172	0.129	0.386	0.322	0.126	1.014	2.510	2.190	1.302	1.441	0.010
Cambodia	+3	0.110	0.927	0.24	0.15	0.28	0.92	0.44	0.260	0.302	0.462	0.508	0.483	0.337	0.359	0.176	0.066	0.276	0.436	0.100	1.580	2.370	1.984	1.067	1.541	0.034
Cameroon	-1	0.083	0.785	0.10	0.16	0.32	0.47	0.85	0.306	0.355	0.661	0.301	0.626	0.333	0.325	0.113	0.176	0.150	0.328	0.262	2.102	2.486	2.112	1.816	1.008	0.025
Congo, Rep.	-	0.059	0.824	0.21	0.14	0.08	0.67	0.79	0.303	0.329	0.538	0.319	0.717	0.316	0.370	0.164	0.166	0.090	0.314	0.159	1.606	2.364	2.424	1.526	1.190	0.012
DR Congo	-	0.093	0.916	0.22	0.24	0.17	0.70	0.84	0.279	0.412	0.563	0.371	0.679	0.345	0.330	0.152	0.134	0.143	0.374	0.147	1.757	1.940	2.067	1.376	1.156	0.026
Ethiopia	-	0.269	0.982	0.24	0.25	0.63	0.91	0.98	0.289	0.409	0.724	0.537	0.869	0.364	0.323	0.088	0.065	0.028	0.602	0.064	1.336	1.595	1.311	1.107	1.075	0.134
Ghana	-3	0.062	0.711	0.06	0.06	0.23	0.70	0.60	0.241	0.306	0.666	0.335	0.543	0.316	0.477	0.110	0.155	0.213	0.325	0.144	1.870	2.204	2.133	1.232	1.168	0.014
Haiti	-3	0.123	0.883	0.19	0.11	0.40	0.67	0.85	0.317	0.428	0.635	0.385	0.683	0.351	0.333	0.112	0.128	0.124	0.419	0.147	1.961	2.133	1.673	1.500	1.264	0.044
India	+5	0.108	0.846	0.47	0.26	0.22	0.74	0.68	0.356	0.423	0.679	0.407	0.582	0.249	0.295	0.115	0.116	0.177	0.440	0.134	1.318	2.006	2.198	1.387	1.341	0.045
Kenya	-	0.083	0.887	0.17	0.14	0.14	0.71	0.91	0.276	0.344	0.635	0.399	0.529	0.337	0.410	0.139	0.129	0.236	0.347	0.180	2.179	2.569	2.871	1.595	1.330	0.026
Liberia	-7	0.130	0.904	0.14	0.18	0.33	0.69	0.79	0.238	0.377	0.734	0.370	0.807	0.365	0.358	0.091	0.129	0.043	0.398	0.141	1.646	1.611	1.900	1.288	1.198	0.042
Malawi	+1	0.096	0.951	0.09	0.16	0.30	0.69	0.97	0.257	0.344	0.528	0.384	0.498	0.410	0.435	0.140	0.128	0.310	0.389	0.167	1.974	2.014	2.034	1.448	1.167	0.032
Mali	-	0.205	0.909	0.19	0.32	0.68	0.67	0.62	0.279	0.434	0.797	0.357	0.461	0.345	0.302	0.070	0.142	0.318	0.531	0.090	1.452	1.530	1.285	1.329	1.205	0.089
Moldova	-1	0.007	0.228	0.21	0.03	0.16	0.20	0.59	0.234	0.175	0.391	0.240	0.466	0.363	0.534	0.286	0.264	0.296	0.181	0.249	0.931	0.764	2.863	2.121	1.121	0.001
Morocco	+2	0.058	0.578	0.20	0.10	0.45	0.55	0.50	0.250	0.369	0.578	0.278	0.528	0.312	0.386	0.162	0.205	0.183	0.381	0.139	1.154	2.068	1.598	1.550	1.429	0.017
Mozambique	-	0.135	0.938	0.10	0.19	0.60	0.61	0.96	0.205	0.382	0.573	0.334	0.555	0.431	0.403	0.144	0.156	0.245	0.472	0.133	1.558	1.861	1.417	1.525	1.093	0.054
Namibia	+3	0.048	0.632	0.33	0.18	0.14	0.53	0.60	0.314	0.309	0.584	0.334	0.670	0.327	0.473	0.151	0.189	0.095	0.310	0.192	1.460	2.486	2.664	1.777	1.302	0.012
Nepal	+4	0.113	0.903	0.34	0.29	0.34	0.64	0.86	0.309	0.343	0.629	0.375	0.558	0.290	0.319	0.125	0.138	0.185	0.443	0.173	1.587	2.033	1.904	1.552	1.222	0.051
Niger	-	0.267	0.971	0.21	0.40	0.68	0.77	0.90	0.271	0.460	0.844	0.387	0.739	0.297	0.261	0.049	0.128	0.069	0.595	0.076	1.489	1.413	1.258	1.213	1.064	0.136