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Contract Law and Development

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Abstract

We relate the design of contract law to the process of development. In this paper, contract law defines which private agreements are enforceable (i.e. are binding and enforced by courts) and which are not. Specifically, we consider an economy where agents face a hold-up problem (moral hazard in teams). The resulting time-inconsistency problem leads to inefficiently low levels of effort and trading among agents. The solution to this problem requires a social contract which meets two conditions: (i) an economywide delegate (judge) responsible for the enforcement of the social contract and (ii) a set of non-enforceable private contracts (regulation). However, because this mechanism is costly, its effectiveness depends on the aggregate production of the economy. To capture the interaction between contract enforcement and development, we introduce a multiperiod economy and show that, in the early stages of development, the mechanism is infeasible. The appearance of enforcement institutions and regulation is delayed for the later stages. At this point of time, the hold-up problem is solved and this spurs economic growth further. Finally, the relationship between economic development and the evolution of contract law may be non-monotonic, which may explain why empirical studies fail to find a robust relationship between the two.

Keywords: contract law, development, enforcement institutions, hold-up, institutional agent, regulation, social contract

JEL Classification: D02, D82, D86, K12, O12, O31, O43

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1 Introduction

The main body of the economic literature on contract theory assumes that private agreements are enforceable by an external authority so that private contracts are binding for the contracting members¹. It is also assumed that the enforcement of all agreements, which private parties are willing to commit to, increases social welfare. Because the role of enforcement institutions is obvious in this context, the related papers do not attempt to explicitly model the enforcement authority and how it achieves its goals. On the other hand, the literature on economic growth and development has placed emphasis on the importance of property rights and their enforcement, but contract law has received disproportionately less attention.

This papers relates the emergence and evolution of contract law to the process of development. We show that the relationship between the two is reciprocal. As the process of development unfolds, contract law evolves accordingly and at the same time it generates new opportunities for economic growth and development. Furthermore, the relationship between the two may be non-monotonic and hence it is difficult to be captured by empirical studies. Through the analysis, we also rationalize two stylized facts of enforcement institutions: i) the existence of institutional agents (such as judges or bureaucrats) who act on behalf of these institutions and are rewarded for their function, and ii) the fact that not all types of private agreements are permitted in a society (regulation).

By contract law we mean the types of private agreements which are enforced by judicial institutions as binding contracts. If a type of private transaction is non-enforceable this means that judges will not impose the terms of the contract on the transacting members, even if one of them breaches the initial agreement. Since most agreements made into contracts are time-inconsistent (i.e. party members do not wish to carry out their part of the initial promise when the execution time comes), non-enforceability of certain transactions effectively prohibits them from taking place.

In order to make the points described above, we present a simple economy which consists of multiple pairs of agents. Each pair is comprised by a producer and a consumer of a specialized good, who face a bilateral hold-up problem. The cost of the specialized good is uncertain and trade is valuable to both parts only if the low-cost state materializes, in which case the surplus generated by trade is divided between them according to their bargaining power. The probability of this event depends on the effort levels of both the consumer and the producer. The effort levels, the state of nature and

¹See for example page 3 in the introduction of "Contract Theory" by Bolton and Dewatripont.

the utilities of agents are non-verifiable. Because both agents bear the full marginal cost of effort exertion but receive only part of the marginal benefit, they generally exert sub-optimal effort levels. The problem could be solved by a mechanism of transfers contingent on trade, if agents could commit not to make any other private agreement for transfers of resources. However, this solution is not time-consistent and, as a result, mechanisms with exogenous full enforcement can not implement the first-best outcome.

In the first part of the paper, we propose a mechanism which solves this timeinconsistency problem. In our mechanism, the agents themselves decide which contracts are enforceable and which are not. That is, contract enforcement is endogenous. It turns out that the implementation of the first-best requires that some contracts be non-enforceable. Specifically, the mechanism involves a social contract between an institutional agent, who has the enforcement power in the economy, and the rest of the agents. The social contract specifies what types of private agreements are enforceable or not at each point in time and the rewards and punishments for the institutional agent, if she acts according to its clauses or not. Thus the optimal social contract specifies ex-post transfers from the trading members to the institutional agent or from the institutional agent to the trading parties, conditional on the occurrence of trade.

We show that this mechanism can induce agents to exert optimal effort levels and is renegotiation-proof if: i) at least one agent plays the role of the institutional agent so that she has a stake to the outcomes generated by the enforcement process and ii) some forms of private agreements are not enforceable and hence non-credible. However, the implementation of the institutional design also requires a minimum amount of resources and so it is inevitably related to the level of economic development.

At this point, we should note this is a model of perfect enforcement. That is, there are no exogenous frictions on enforcement. It is costless for the judge to obtain information on verifiable variables and there are no costs for punishing an agent who did not execute her part of the contract. Furthermore, we do not impose any exogenous restrictions on the contract space, apart from the condition that contracts should be written on verifiable variables (the natural one). However, in equilibrium, agents will choose to render some (otherwise fully enforceable) contracts as non-enforceable, in order to credibly commit themselves to exert high-effort. In other words, we abstract away from any possible enforcement friction in order to show that the contractual space is optimally chosen to be incomplete.

In the second part of the paper we introduce a multi-period economy and connect the hold-up problem (and its solution) to the process of economic growth and development. The agents in the economy have access to the production technology of a non-specialized or autarchic good, which exhibits no uncertainty, and to the production technology of a specialized good in pairs of buyer and seller, exactly as in the previous section. The economy starts with no initial institutions, but at the beginning of the first period agents can propose and generate institutions of government and enforcement through a social contract. Institutional agents play the dual role of the governor and judge and they are responsible for imposing taxation to agents, spend public revenues to investments (which increase the productivity of agents), enforce private agreements and provide reductions in taxation conditional on the trade of the specialized good. The reductions in taxation work as subsidies which induce agents to exert higher effort and thus increase the value of trade.

In this economic set-up, the initial level and the rate of increase of productivity in the production of the autarchic good are the main parameters which drive the evolution of institutions and growth. The most interesting case is when productivity growth is fast enough so that the economy passes from multiple stages of development. We identify the necessary conditions for this case to arise and we derive the following results: i) The economy starts from low levels of productivity, where the trade of the specialized good is not feasible, but as productivity increases the feasibility constraints are relaxed. At a specific point in time trade becomes feasible and enforcement institutions are created. ii) Restrictions on the set of enforceable private agreements arise endogenously, inducing trading parties to exert high effort and to increase the value of trade and this spurs further economic growth. Therefore, the causal relationship between enforcement institutions and growth goes in both directions. iii) The change of restrictions on the set of enforceable agreements is non-monotonic to the process of economic growth.

We believe that these results are interesting because they emphasize the reciprocal relationship between contract law and growth. More importantly, the relationship between the two may be non-monotonic. Accemoglu and Johnson (2005) test whether property rights institutions or contract enforcement institutions have a positive impact on growth. While they find that property rights seem indeed to affect growth positively, contract institutions do not present a statistically significant impact. Their main hypothesis is that the lower the cost of contract enforcement, the more easily private parties can contract and hence the greater the impact of economic growth. They test for this effect by using as a proxy for enforcement costs the legal origin of contract law (whether it is common law or civil law).

While, our model can not distinguish between the two types of law origins, if one is willing to assume that enforcement costs are positively correlated with the number of transactions which are enforceable (which we justify in section 4, in the part where we discuss the empirical implications of the model), then we can offer an explanation to the findings of the empirical literature. With a non-monotonic relationship between the set of enforceable agreements and the level of growth, the impact of contract law on the latter can not be captured by linear specifications.

Our results also show that regulation may be optimal for social welfare. Thus, limiting agents' economic freedom may have beneficial results if hold-up problems are prevalent in economic exchanges, a point which goes against the classic economic intuition that more economic freedom implies greater welfare. Apart from the example of trade of specialized goods, which we provide in this paper, there are many other cases of economic interest, where a trade-off between ex-ante incentives and ex-post efficiency may arise. Our framework can be applied in these cases to explain why certain types of regulation are imposed or why certain types of contracts are forbidden from being written.

For instance, our model could be used to explain the abolishment of slavery in the 19th century as banning certain property contracts in order to induce the accumulation of unobservable human capital. It could also be used to rationalize certain laws, which protect collective bargaining agreements between employees and employers as an effective banning of one-to-one contracts. This, in turn, increases labour wages and, apart from redistributive effects, it increases the ex-ante incentives of workers to acquire human capital. On the other hand, forbidding prenuptial agreements, which may dissolve dysfunctional partnerships in an effective way ex-post, may be a way into incentivizing parents in exerting high effort to their family affairs and children up-bringing.

We also believe that our model can be extended to generate a more complete theory of the trade-off between economic freedom and incentives (ex-post efficiency versus exante incentives) and it can also provide a model of regulation cycles. Finally, we provide a different rationale for the existence of institutional agents, such as bureaucrats and judges. Besides being the executors of authority, they also guarantee the credibility of the institutions they represent by having a stake in their functionality and by not always agreeing to change them. Thus institutions acquire persistence, which again is crucial for their credibility and the solution of hold-up problems.

2 Related Literature

There is an extensive literature dealing with the determinants of property rights, their value for society and development. Examples include Umbeck (1981), Skaperdas (1982), Grossman (2001) and Gonzalez (2007). On the contrary, there is little work on how en-

forcement institutions and contract law can affect the development process. A notable exception is Dhillon and Rigolini (2009), which relates the process of development to enforcement institutions through the functioning of commodity markets. Our framework differs from theirs in two ways. First, we consider only formal institutions while their model is concerned with the co-determination of both formal and informal institutions of enforcement. Second, the process of development in their paper is exogenous and related to the reliability of the production process to generate high quality goods, while we are concerned with the co-evolution of contract law and development through the changes on regulation and productivity respectively.

A different strand of literature examines the impact of limited enforcement on economic transactions. Telser (1980) is one of the first papers to model self-enforcing agreements, while Bull (1987) examines self-enforcing agreements in the context of the US labor market and Ray (2002) examines their time-structure. However, these papers are concerned with cases where contract enforcement is impossible and this is an economic restriction which agents can not overcome. Other papers, like Cooley, Marimon and Quadrini (2004) and Ellingsen and Kristiansen (2008), are concerned with the impact of limited enforceability on financial contracting. Krasa and Villamil (2000) consider the case where enforcement of the contractual agreement is a choice variable of the contracting members but it is costly and show that the costly state verification model can be seen as a reduced form of their enforcement problem. None of the above papers, however, examines the issue of enforceability from the perspective of endogenous limitations on the types of agreements that are enforced.

On the other hand, there is an extensive literature that is concerned with the issues of institutional authority. Aghion and Tirole (1997), Aghion, Alesina and Trebbi (2004), Greif and Laitin (2004), Greif et al (2008), Laffont and Martimort (1998), Sanchez and Straub (2006) are some of the papers concerned with the issues of authority in organizations or the endogenous formation of institutions. Our main difference is that we focus on enforcement institutions and its relationship to the process of economic development. On top of that, in our analysis we combine both the questions of how these institutions emerge and evolve and how authority is determined.

Our paper is also related to the literature regarding the hold-up problem. Since the seminal contributions by Grossman and Hart (1986) and Hart and Moore (1988, 1990), a long list of papers has been devoted to presenting the inefficiencies generated by this problem or solving it². We do not attempt to solve the most general type of hold-up

²See for example Aghion, Dewatripont and Rey (1994), Che and Hausch (1999), Hart and Moore (1999), Maskin and Moore (1999), Maskin and Tirole (1999a), Maskin and Tirole (1999b), Guriev and

in this paper. We use a simple example of a hold-up problem (which can not be solved by the mechanisms presented in the papers above) in order to show the importance of enforcement institutions in solving time-inconsistency problems.

However, Baliga and Sjöström (2009) adopt a very similar framework to ours. In their paper they allow agents to contract with a third party and show that, with an appropriately designed mechanism, agents can solve their time-inconsistency problem. Furthermore, they show that side-contracting between the third party and one of the agents does not alter their results. They apply their framework to the hold-up problem and to the problem of moral-hazard in teams. We obtain similar results through the institutional agent. The main differences are two. First, the third party in our paper has a specific type of authority in the economy (to enforce private agreements or not), which the third party in their paper does not have. Second, and most important, they assume that all private agreements are enforceable while we treat enforceability as an endogenous variable.

Finally, our paper is related to the literature concerned with issues of delegation. These papers examine the ability of an uninformed principal to extract information from an informed agent, who is asked to perform a task. Some examples are the papers by Holmström (1982), Fershtman, Judd and Kalai (1991), Faure-Grimaud, Laffont and Martimort (2003), Szalay (2005) and Alonso and Matouschek (2008). While in these papers delegates act on behalf of a principal, in our model institutional agents are economy-wide delegates who act on behalf of the society. We also eschew away from issues of information extraction as we assume that the actions of institutional agents are fully observable by the rest of the agents.

3 A simple model with two agents

In this section we examine the solution to a static hold-up problem and derive the main results and intuition which are required for the analysis of the dynamic model.

A plough-maker (say agent j, who is also sometimes referred as the producer) and a farmer (agent i) face a simple hold-up problem with bilateral externalities. The plough-maker has the ability to produce one unit of plough g, which is custom-made to satisfy the farmer's requirements. This good has valuation equal to v for the farmer in terms of some numeraire commodity (think of it as the additional quantity of wheat, which the farmer can produce due to its use).

Kvassov (2005) and Evans (2008)

However, the cost of the plough, which is again reflected in terms of the numeraire commodity, is uncertain and depends on the state of nature. There are two states of nature, one with a high cost ($\theta_1 : k_1 = k_H$) and one with a low cost ($\theta_2 : k_2 = k_L$). The probability of the low-cost state depends on the effort level of both the producer and the farmer. The intuition for this assumption is that effort is exerted for acquiring skills relevant to their occupation. The more skilled the farmer is in cultivating the land, the higher the chances that he requires a crude, but inexpensive plough to do his job. Similarly, the more skilled the plough-maker is, the higher the chances that he can produced the required plough with minimal use of resources³.

If e_i and e_j are the effort levels exerted by them, then $f(e_i, e_j)$ is the joint probability function of the low cost state arising. There are two effort levels for each agent: $\{\overline{e}, \underline{e}\}$: $\overline{e} > \underline{e}$. The corresponding cost of effort, which is homogeneous across agents, is given by: $c_i(\overline{e}) = c_j(\overline{e}) = \overline{c} > c_i(\underline{e}) = c_j(\underline{e}) = \underline{c}$ and $f(\overline{e}, \overline{e}) > f(\overline{e}, \underline{e}) = f(\underline{e}, \overline{e}) > f(\underline{e}, \underline{e})$. The probability function f exhibits decreasing returns to scale: $f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e}) > f(\overline{e}, \underline{e}) = f(\underline{e}, \underline{$

The agents are endowed with a sufficiently large amount of the numeraire commodity w (think of it as wheat or bread)⁴. Moreover, agents are risk neutral. The utility of the plough-maker depends only on the amount of the numeraire commodity that she consumes, but the utility of the farmer also depends on good g. In terms of notation, $u_j = x_j^a - c_j$ and $u_i = x_i^a + I_g v - c_i$, where x_j^a and x_i^a are the consumption levels of the numeraire commodity by the producer and the farmer respectively and I_g is an indicator function that takes the value one if g is traded and zero otherwise. If agents decide to trade after the state of nature is realized, they divide the gains from trade according to some exogenous bargaining power⁵. Let β_i (β_j) be the bargaining power of agent i (j), with $0 < \beta_i < 1$ and $\beta_i + \beta_j = 1$. Furthermore, assume that the following inequalities hold:

³Alternatively, the model can be formulated so that uncertainty is added on the consumption side of the economy. For instance, the valuation for the plough could be high or low and its probability can depend on the effort level of the farmer. In this case, there would be four states of nature, but the main results of the paper would be the same if the trade generates social surplus in only one of the four states. The economic interpretation would be similar as well, especially if we think of goods which require some effort from the farmer's part in order to learn how to use them efficiently.

⁴ for the purposes of this section it is sufficient that $w > v - k_L$

⁵This can be interpreted as the probability of one side to make a take-it-or-leave-it offer to the other side.

$$[f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})]\beta_{\xi}(v - k_L) - (\overline{c} - \underline{c}) < 0, \forall e_{\zeta} \in \{\overline{e}, \underline{e}\}$$
(1)

$$[f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})](v - k_L) - (\overline{c} - \underline{c}) > 0, \forall e_{\zeta} \in \{\overline{e}, \underline{e}\}$$
(2)

$$k_l - k_H + 2 \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} > 0$$
(3)

In the inequalities above, ξ denotes one of the two parts of the transaction (either *i* or *j*) and ζ denotes the other part (if $\xi = i$, then $\zeta = j$ and vice versa). The timing of events is: first, agents choose effort levels, then the state of nature is determined and then agents decide whether to trade or not. Committing to trade before uncertainty is resolved is sub-optimal as it entails welfare losses in the state where the cost of *g* is higher than *v*.

Inequalities (1) and (2) reflect the conditions under which the hold-up problem arises. Inequality (1) states that, independent of the effort level of the other agent, the marginal benefit of increasing the effort level for type ξ is lower than the marginal cost of doing so, while (2) states that, under the same conditions, the marginal social benefit exceeds the marginal social cost, which is equal to the private cost. In conjunction, inequalities (1) and (2) imply that it is a dominant strategy for both types of agents to exert low effort, but this is socially sub-optimal.

In fact, adding inequality (2) for different effort levels and types, one can show that $[f(\overline{e}, \overline{e}) - f(\underline{e}, \underline{e})](v - k_L) - 2(\overline{c} - \underline{c}) > 0$, which means that if both agents choose to exert high effort the aggregate social benefit is positive. In other words, the two conditions form a simple model of a hold-up problem: both agents would benefit from exerting high effort but because they receive only a part of the social surplus they generate, they do not have an incentive to do so.

Inequality (3), on the other hand, is a technical condition, which is required for making the incentive compatibility problem meaningful. As we show in the proof of Proposition 1, when this condition holds, subsidizing trade in order to induce high effort, without any restrictions on enforceable agreements, is ineffective, because agents prefer to trade in the high cost in order to receive the subsidies and redistribute them among themselves.

Effort levels, effort costs, the state of nature and the level of utility of each agent are observable but non-verifiable. In addition, the effort levels of the agents are nontransferable, so that property rights can not solve the problem. This is a plausible assumption since in many cases of economic interest the economic surplus may depend on the actions of some individuals which can not be easily replicated by others. In our example, the plough-maker has a specific set of skills which are needed for the production of the good, which the farmer does not have and can not acquire.

If effort choices were transferable, an easy solution to this problem would be the allocation and trade of property rights on the effort decisions (see also Grossman and Hart, 1986). Also, if any of the non-verifiable variables, could be verified, even at some cost, then the agents could design mechanisms of subsidy provision or punishments in order to induce the first-best effort levels. For example, if the state of nature were costlessly verifiable, then the following mechanism would implement the first-best effort levels: the farmer and the plough-maker give out $\tau_i = f(\bar{e}, \bar{e})(1 - \beta_i)(v - k_L)$ and $\tau_j = f(\bar{e}, \bar{e})\beta_i(v - k_L)$ units of the numeraire commodity to a risk-neutral agent, who has the obligation to return to them $s_i = (1 - \beta_i)(v - k_L)$ and $s_j = \beta_i(v - k_L)$ units respectively, if the low-cost state arises. In such a case, agents receive a subsidy in the low-cost case which aligns the marginal costs and benefits of effort exertion to the social costs and benefits and therefore achieves first-best outcomes.

Of course, as the relevant literature points out (see for example Hart and Moore (1988) or Maskin and Tirole (1999b)), the problem with such a mechanism is that truthtelling about the state of nature is not incentive compatible when the states of nature are non-verifiable. If the high-cost state arises, agents have incentive to lie in order to receive and redistribute the subsidies between them. Such redistributions require binding agreements on net transfers of resources. Ex-ante, agents prefer to ban such transfers, so that the mechanism satisfies incentive compatibility and generates optimal incentives for effort provision, but ex-post agents prefer to renegotiate the mechanism and allow for the transfers to take place. Therefore, this mechanism fails to provide incentives for efficient effort exertion, because it is not renegotiation-proof and, hence, credible.

This section of the paper shows how this problem can be circumvented by the introduction of an **institutional agent**, a type of delegate, who enforces the mechanism and whose final payoff depends on the outcome of the mechanism. In this case, because the institutional agent has an incentive to block any renegotiation that reduces his expected payoff, we show that an endogenous commitment not-to-renegotiate the mechanism arises ex-post. Furthermore, the enforceability of ex-post transfers and the incentives of the institutional agent arise endogenously, through the ex-ante **social contract** between the agents and their delegate.

For the rest of this section we assume the timing of events as represented by Figure

1, which is similar to the one adopted by Watson (2007). Before proceeding to the main result of the section, we provide the necessary definitions.



Figure 1: Timing of Events

Definition 1: An institutional agent ($\Sigma = 1$) is a third party whose actions and rewards are determined by the agents through a social contract. The action set of the institutional agent is the payment of subsidies to the agents and the enforcement or not of private contracts.

Definition 2: A private contract $\pi(q, p(g), I_g)$ is any agreement between the farmer and the plough-maker. This formulation includes agreements for selling good g at a price p(g), agreements which promise a net transfer of resources q conditional on the trade and the price of good g (side contracts) and agreements for an unconditional transfer of resources q (irrespectively of trade). Q is the set of all possible contracts. Whether a subset of Q is enforceable or not depends on the social contract.

Definition 3: A social contract $S(\Sigma, \Phi(Q), \tau)$ is a contract between the farmer,

the plough-maker and potentially (but not necessarily) an institutional agent, which defines ex-post transfers τ conditional on the verifiable trade of the good g and on its price $p(g)^6$, the inclusion of the institutional agent or not ($\Sigma = 1$ or $\Sigma = 0$, respectively) and the set of enforceable private agreements $\Phi(Q)$.

We assume that even if an institutional agent is not included in the social contract, the farmer and the plough-maker can still utilize the enforcement authority of the economy (which we treat as an automaton or a machine in that case). We do this so that we can contrast our results with the existing literature, which assumes that enforcement authorities exist but they are not explicitly modeled. In particular, we want to show why the incompleteness of the contractual space and the structure of incentives for the institutional agent as well, are so important for solving the hold-up problem.

However, in the analysis that follows we implicitly assume that the judge is punished if he violates his part of the agreement with the other two agents, and hence he is bound to execute the social contract, unless they all agree to renegotiate it⁷. Also notice that the cases of exogenous enforcement with complete or incomplete set of enforceable agreements are special cases of social contracts in our framework. In section 4, we make the more realistic assumption that the economy starts of from a point of no institutions and we derive their emergence, evolution and structure endogenously.

Propositions 1 and 2 below show the role of contract law for solving the hold-up problem. The main intuition is that, in order to induce agents to exert high effort, subsidization of trade is required. However, once the high-cost state materializes, agents may have an incentive to conduct trade, even if it is suboptimal, in order to receive the subsidies. Stopping them from doing so requires limitations to the maximum amount of net transfers that the farmer can provide to the producer. However, these limitations, though optimal from an ex-ante point of view, are not credible ex-post. Without an institutional agent, who bears the cost of subsidies and the benefits of taxation, the agents would simply undo the regulation they set in place after the state of nature realizes. Therefore, a credible solution to this time-inconsistency problem requires a

⁶Ex-post transfers can be either positive (subsidization) or negative (taxation).

⁷Of course, the important question of what ensures the compliance of the institutional agent with the social contract is omitted from our analysis. However, one way to deal with this issue is to assume an infinitely lived judge and construct a reward structure for him and trigger-strategies for the rest of the agents, such that the compliance with the social contract is self-enforced. We leave this interesting direction for future work.

judge, who enforces regulation and who has a stake on the function of the enforcement institutions. In other words, the solution to the hold-up problem is not imposed exogenously, but arises as the equilibrium outcome of an institutional process (the social contract).

Proposition 1: Let $\overline{p} = k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})}$. Consider the social contract S^* , which defines:

i) if trade of good g takes place, agent ξ receives subsidy (negative taxation) $\tau_{\xi 1} = (1 - f(\overline{e}, \overline{e})) \left(\beta_{\xi}(v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) < 0.$ ii) if trade of good g does not takes place, agent ξ pays out taxation $\tau_{\xi 0} = f(\overline{e}, \overline{e}) \left(-\beta_{\xi}(v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) > 0.$ iii) any private contract $\pi(0, \hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is nonenforceable.

Then S^* implements the first best effort levels and it is renegotiation-proof.

Proposition 2: The existence of the institutional agent and the non-enforcement of private contracts contingent on trade are necessary conditions for the implementation of first-best effort levels.

The proofs of the propositions are provided in Appendix I. Given these results, one can also see the implications of this mechanism for development. Social contracts can solve the hold-up problem as long as there is sufficient production to be taxed and provided as subsidies. At the early stages of development, production is relatively low and the mechanism is infeasible. As a result, there is low effort exertion and low probability of trade. Once productivity increases sufficiently, then the mechanism becomes feasible and this implies the emergence of regulation, which is necessary to support high effort levels. In turn, the probability and the marginal value of trade increase, which encourages more investments to the know-how of the specialized good and promotes productivity growth further. Therefore, a feedback mechanism emerges between economic growth and enforcement institutions. The following section formalizes these arguments and shows why the relationship between the two may be non-monotonic.

4 Enforcement Institutions and Development

This section links the process of economic development with contract law. Formally, the model follows closely the notation and assumptions of the previous section. The economy lasts for T periods and consists of two groups of agents, each one of which is represented by a continuum of measure one. The group i are buyers (farmers) of the specialized good and group j are sellers (plough-makers). Each seller j can produce a specific variety of an intermediary good g, which a specific buyer i can use to generate additional production of the numeraire commodity (good a). In each period, a random matching process matches one agent from group i with one from group j and together they form an exclusive partnership⁸. Apart from their specialization in a specific variety of g, the partnerships are identical and thus we use i and j as the notation for the representative buyer-seller party.

The structure of the production for the specialized good is the same as in section 3, but we allow the gains from trade to vary over time by a scaling factor Γ_t^{9} . Γ_t is a positive coefficient, common for the whole economy, which can be interpreted as the productivity on agents' effort levels for good g. Agents can save resources (in terms of the numeraire commodity) and invest in an aggregate, non-depreciable amount of capital Z_g which increases the value of trade Γ_t . We assume that Γ_t is a concave function of Z_g .

In addition to the specialized good g, both agents have access to the production technology of the numeraire commodity a, which can be thought of as a non-specialized or "autarchic" good (we refer to any these terms interchangeably). The output of this good in each period is a linear non-stochastic function of the economy-wide productivity variable A_t and of the effort level an agent exerts for its production: $y_{\xi t} = A_t e_{\xi at}$, where $y_{\xi t}$ is the production of good a in period t by an agent of type ξ and $e_{\xi at}$ is the effort level exerted by her¹⁰. Unlike the effort levels for the production of good g, we assume that the effort level for the production of good a is continuous and the cost of effort $c_{\xi at}$ is a convex function of $e_{\xi at}$ with $\frac{\partial c_{\xi at}}{\partial e_{\xi at}} > 0$, $\frac{\partial^2 c_{\xi at}}{(\partial e_{\xi at})^2} > 0$. The total cost of effort for an agent is the summation of the two efforts: $c_{\xi t} = c_{\xi at} + c_{\xi gt}$.

The productivity variables A_t and Γ_t are common for all agents. Furthermore, their values depend on two types of cumulative, non-depreciable capital, Z_a and Z_g respectively. These capital levels can be interpreted as the technological know-how of the economy for the production of the non-specialized and the specialized good

⁸This means that the buyer has the know-how of using only a specific variety of the good g, which is produced by only a specific seller and the appropriate variety changes randomly every year.

⁹So, good g yields $\Gamma_t v$ units of good a to the appropriate buyer, its cost takes values $\Gamma_t k_H$ or $\Gamma_t k_L$ and the cost of effort by an agent ξ for g in period t is $\Gamma_t c_{\xi gt}$. Also, conditions (1) and (2) hold.

¹⁰*i* has an increased production of good *a* if she buys the appropriate variety of the specialized good. This is additional to y_{iat} .

respectively¹¹. Agents can choose to save amounts of the non-specialized good and invest into the two forms of capital. However, due to the infinitesimal size each agent and the fact that productivities are common for all, private savings are zero. Hence, we ignore private savings in the analysis and consider public taxation and investment, conducted by a governor.

Let z_{at} , z_{gt} be the aggregate investment to the production process a and g respectively in period t. We also assume that $A_t(Z_a)$ and $\Gamma_t(Z_g)$ are concave functions of the respective capital stocks, satisfying the following conditions: A(0) > 0, $\Gamma(0) > 0$, Inada condition: $A'(0) = \Gamma'(0) = \infty$. Assuming that the economy starts with zero capital stocks ($Z_{a0} = Z_{g0} = 0$), the first two conditions guarantee that some production is attainable even with zero capital stocks while the last two conditions guarantee that at least some investment in productivity is socially beneficial.

Agents are risk-neutral. The state-dependent utility is the summation of production of a, gains from trade, taxation and cost of effort:

$$u_{i\theta t} = y_{it} + I_g(\Gamma_t v - p) - \tau_{igt} - (c_{iat} + c_{igt})$$
$$u_{j\theta t} = y_{jt} + I_g(p - k_\theta) - \tau_{jgt} - (c_{jat} + c_{jgt})$$
$$U_{\xi} = \sum_{t=0}^T \delta^t \left(\sum_{\theta} f_{\theta}(e_{\xi gt}, e_{\zeta gt}) u_{\xi \theta t} \right)$$

In terms of the timing of events, every period is split into sub-periods which roughly follow the order of events of section 3. In the beginning of period zero the agents of the economy propose and vote social contracts, which establish the main institutions of government and enforcement in the economy. Thereafter, in every period t, agents decide how much effort to exert on autarchic and specialized production, the state of nature is determined, and agents decide whether to trade or not and sign private contracts. Once production (and potentially trade) has taken place, the institutional agent imposes taxation (conditional on trade) and decides whether to enforce certain private agreements. Finally, consumption takes place. Figure 2 presents the sub-stages of the game and the timing of events for every period after period zero.

The social contract S is similar to section 3, with three main differences: i) Given that it is proposed in period 0, it is an exhaustive plan of all future dates. ii) It includes investment plans z_a , z_g for increasing respective productivities A and Γ , so S takes now

¹¹Alternatively, we could allow the capital levels to also depend on effort and think of the relative cost in terms of foregone production, but the mathematical formulation of the two problems is identical.

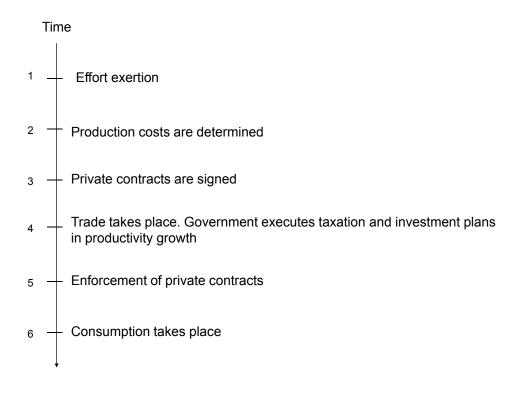


Figure 2: Timing of Events

the form: $S(\Sigma, \Phi(Q), \tau, z_a, z_g)$. iii) Σ can contain multiple agents. In this context, an institutional agent can be thought of as a governor and a judge at the same time (there is no separation of powers).

As mentioned above, in the beginning of the game any agent of the economy can propose a social contract. A proposal becomes a valid social contract if all agents vote for it (unanimity requirement), even if there is only one proposal made¹². If no proposal achieves the unanimity requirement, then there is no government and enforcement institutions in the economy, and agents can utilize only the autarchic production technology. In this case, the game proceeds with effort exertion, production and consumption under autarchy in each period¹³.

Also notice that the unanimity requirement for the selection of the social contract acts as a participation constraint. If the enforcement of a proposed social contract gives

 $^{^{12}\}mathrm{This}$ means that agents have the option of not voting at all.

¹³The state of nature is inconsequential for autarchic production and therefore it is omitted from the analysis of these sub-games

lower utility to an agent than the utility of autarchy, then the agent can block the social contract by not voting for it. Therefore, there can be no equilibrium of the game where the final expected utility for a subset of agents is below their autarchic continuation utility. For the rest of the section, when we refer to participation constraints, we imply the autarchic utility level that each agent receives in each period.

On the other hand, any proposal that generates a Pareto undominated allocation and satisfies the participation constraints of all agents can be an equilibrium of the game. This is again a result of the unanimity requirement, since it gives veto power to players¹⁴. In the analysis of the following sub-sections we analyze only one of the equilibria of the game, the equilibrium where the proposed social contract is designed so as to maximize the summation of the utilities of all non-institutional agents, given that it satisfies the participation constraint of the institutional agents¹⁵.

If a certain social contract is voted in the beginning period zero, the selected institutional agents do not exert effort in the production process. In other words, we assume that the activities of the institutional agent and production are mutually exclusive¹⁶. The rest of the agents decide on how much effort to exert in producing the generic good or investing effort for the specialized good and whether to trade or not.

Before proceeding to the analysis of the problem, it is worth mentioning that institutional agents play a dual role in this model. They are both governors and judges at the same time. They are governors because they collect taxes from the rest of the agents and allocate them to public spending, i.e. investments to the two types of productivities. They are also judges because the enforce private agreements. This dual role seems to better fit the role of monarchs in pre-industrial economies. In this paper we do not examine the potential reasons behind the separation of powers in post-industrial economies. Our main focus are the questions of when do restrictions on enforcement arise and how they affect the development path of the economy.

¹⁴The analysis of this result is available from the author upon request.

¹⁵We effectively compute the social contract which arises from a Nash-bargaining procedure which allocates an equal bargaining coefficient on both types. The general set of equilibria, with unrestricted coefficients can also be derived, however, we simplify the analysis by examining only the limiting case of equal bargaining power.

¹⁶One consequence of this assumption is that occupation in institutions generates a cost in terms of foregone production and hence a trade-off between the functionality of institutions and productive capacity. It also implies a simple solution to this trade-off: the minimization of the set of institutional agents. Relaxing this assumption can generate a more interesting trade-off and richer model predictions for institutions but it would lead us astray from the main topic of the paper. We leave this aspect of the problem for future work.

4.1 Agents' Maximization Problem and Best-Response Functions

An agent of type ξ , who is occupied with production activities, chooses effort levels $e_{\xi gt}$ and $e_{\xi at}$ in each period t in order to maximize his utility, given an accepted social contract S (and the implied values of $\tau_{\xi gt}$, A_t , Γ_t). Each pair (i, j) also exchange g on the "fair" price $p^* = \Gamma_t((1 - \beta_i)v + \beta_i k_L)$, if its cost is low or if the contract law does not allow (side-)payments which violate incentive compatibility. Otherwise, they exchange good g for the price: p+q. Assume the former case¹⁷. In this case agent ξ solves:

$$\max_{\{e_{\xi at}, e_{\xi gt}\}} A(Z_{at-1}) e_{\xi at} + f(e_{igt}, e_{jgt}) \left(\Gamma(Z_{gt-1}) \beta_{\xi}(v - k_L) - \tau_{\xi 1t} \right) + \left(1 - f(e_{igt}, e_{jgt}) \left(-\tau_{\xi 0t} \right) \right) \left(-\Gamma(Z_{gt-1}) c_{igt} - c_{iat} \right)$$

$$(4)$$

The solution to problem (4) is given by:

$$e_{\xi at} : A(Z_{at-1}) = \frac{\partial c_{\xi at}}{\partial e_{\xi at}}$$
(5)

$$\begin{cases} e_{\xi g t} = \overline{e} & \text{if } \tau_{\xi 0 t} - \tau_{\xi 1 t} \geqslant -\frac{\Gamma(Z_{g t-1}) \left[\left(f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta}) \right) \beta_{\xi}(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})} \\ e_{\xi g t} = \underline{e} & \text{otherwise} \end{cases}$$
(6)

Since the value of of $e_{\xi gt}$ also depends on the choice of effort of the trade-partner ζ , equations (5) and (6) give the best-response function of agent ξ .

As far as institutional agents are concerned, they do not directly engage in productive activities. Their utility is a function of the total taxes they collect minus the resources they invest in Z_a , Z_g . Though we do not model it explicitly in the social contract, one can ensure the compliance of institutional agents by including certain rewards and punishments, conditional on its execution or not.

Denote by r_{σ}^+ the reward of an institutional agent for executing the social contract and by r_{σ}^- her punishment if she deviates. Her reward is equal to the aggregate taxes she collects minus the aggregate investments in physical capitals:

$$r_{\sigma}^{+} = f(e_i, e_j)(m_i \tau_{\sigma i1t} + m_j \tau_{\sigma j1t}) + (1 - f(e_i, e_j))(m_i \tau_{\sigma i0t} + m_j \tau_{\sigma j0t}) - z_{\sigma at} - z_{\sigma gt}$$

¹⁷We will shortly show a similar condition to condition iii) of Proposition 1, which ensures incentive compatibility and the "fair" price.

In the expression above, m_i , m_j is the total mass of agents of type *i* and *j* respectively, who are occupied in productive activities (non-institutional agents) and σ is the institutional agent, who is asked to execute the taxation plan τ_{σ} and investment plans $z_{\sigma a}$, $z_{\sigma g}$. On the other hand, if the institutional agent executes different plans in period t, say $\{\tau'_{\sigma\xi\theta t}, z'_{\sigma at}, z'_{\sigma gt}\}$, for $\xi \in \{i, j\}$, for $\theta \in \{0, 1\}$, then her utility is the aggregate units of the autarchic goods she accumulates according to her plan minus her punishment. By adding and subtracting r_{σ}^+ and by using the equation above, we get:

$$f(e_i, e_j) \left(m_i (\tau'_{\sigma i1t} - \tau_{\sigma i1t}) + m_j (\tau'_{\sigma j1t} - \tau_{\sigma j1t}) \right) + (1 - f(e_i, e_j)) \left(m_i (\tau'_{\sigma i0t} - \tau_{\sigma i0t}) + m_j (\tau'_{\sigma j0t} - \tau_{\sigma j0t}) \right) - (z'_{\sigma at} - z_{\sigma at}) - (z'_{\sigma gt} - z_{\sigma gt}) + r_{\sigma}^+ - r_{\sigma}^-.$$

This states that the utility of the institutional agent is the utility she would receive by executing the policy plans of the social contract, plus the extra resources of the new taxation, minus the additional expenses by the new investment plans minus the penalty defined by the social contract for deviating from the original agreement. It is clear that as long as the punishment for deviation is lower than the net additional resources, the institutional agent will choose to deviate.

In fact, the optimal choice of the institutional agent is to tax all production of the non-specialized good so as to maximize the net resources from deviation. Such a behavior can be prevented, of course, by setting the penalty of deviation sufficiently high. Lemma 2 in Appendix I provides the necessary rewards and punishments for an institutional agent, so that her participation constraint is satisfied, while maintaining her incentives for executing the social contract. For the rest of the analysis we suppress this problem from the analysis.

4.2 The Optimal Design of Contract Law

Lemma 1 characterizes the optimal set of institutional agents. Proposition 3 characterizes the optimal design of contract law and follows from Proposition 1. Both proofs are provided in Appendix I.

Lemma 1: The optimal social contract S^* determines that the set Σ is of measure zero. That is, the total number of institutional agents is infinitesimal compared to the aggregate population.

The intuition of Lemma 1 is simple. Since productive and government activities are mutually exclusive and there is no limit to the span of control in governance, having a strictly positive measure of institutional agents is a social waste of resources: it reduces the productive capacity of the economy without generating any additional benefit. Therefore, the minimization of Σ is the optimal option. In other words, the optimal social contract defines that the number of institutional agents is finite, so that some institutional agents exist. However, the exact number is indeterminate, as they do not impact the aggregate economy due to the continuum-of-agents assumption. Without loss of generality, henceforth we assume that there is only one institutional agent in equilibrium.

Proposition 3: Let $\overline{p} = \Gamma(Z_{gt-1}) \left(k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(e_i, \overline{e}) - \overline{f}(e_i, \underline{e})} \right)$. Incentive compatibility requires that any private contract $\pi(0, \hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is non-enforceable.

Proof: See Appendix I

Proposition 3 gives the necessary regulation required to support an incentive compatible subsidization scheme, which parallels that of Proposition 1. Notice that \overline{p} , which stands for the maximum transfer allowed between the two agents when trade takes place, depends on the values of Γ_t , the productivity of the specialized good, and the induced effort level e_i . These are endogenous variables which change over time and, therefore, regulation is dynamic. While the impact of Γ_t has a positive effect on \overline{p} (meaning that it relaxes the incentive compatibility condition), the evolution of e_i has a negative impact on \overline{p} , and the two effects generate a non-monotonic relationship between the optimal contract law and development. We analyze this effect in more detail in section 4.4.

4.3 Unconstrained optimal taxation and investment plans

A consequence of the result stated in the Lemma 1 is that the utility of the institutional agent is infinitesimally small compared to the utility of the agents employed in productive activities and hence it can be ignored in the determination of the optimal social contract. Provided the optimal values for Σ and $\Phi(Q)$ (by Lemma 1 and Proposition 3), the rest of the variables included in the social contract are given as the solution to the problem which maximizes the expected utility of both types of agents with respect to taxation and investment plans and subject to the incentive compatibility, government budget and feasibility constraints:

$$\max \int \left(\sum_{t=0}^{T} \delta^{t} \left(\sum_{\theta} f_{\theta}(e_{\xi g t}, e_{\zeta g t}) u_{it} \right) \right) di + \int \left(\sum_{t=0}^{T} \delta^{t} \left(\sum_{\theta} f_{\theta}(e_{\xi g t}, e_{\zeta g t}) u_{jt} \right) \right) dj \Leftrightarrow$$

$$\max \quad \frac{1}{2} \sum_{t=0}^{T} \left[\delta^{t} \left[A(Z_{at-1}) e_{iat}^{*} + f(e_{igt}, e_{jgt}) \left(\Gamma(Z_{gt-1}) \beta_{i}(v - k_{L}) - \tau_{i1t} \right) + \left(1 - f(e_{igt}, e_{jgt}) \right) \left(-\tau_{i0t} \right) - c_{iat} - c_{igt} \right] \right]$$

$$+ \frac{1}{2} \sum_{t=0}^{T} \left[\delta^{t} \left[A(Z_{at-1}) e_{jat}^{*} + f(e_{igt}, e_{jgt}) \left(\Gamma(Z_{gt-1}) \beta_{j}(v - k_{L}) - \tau_{j1t} \right) + \left(1 - f(e_{igt}, e_{jgt}) \right) \left(-\tau_{j0t} \right) - c_{jat} - c_{jgt} \right] \right]$$
(7)

With respect to: $\{\tau, z_a, z_g\}$, and subject to:

Best Response Function of type j

$$\tau_{j0t} - \tau_{j1t} \leq \Gamma(Z_{gt-1})k_H - (p+q)$$
 Incentive Compatibility Constraint for type j (10)

$$z_{at} + z_{gt} \leq f(e_{igt}, e_{jgt})(\tau_{i1t} + \tau_{j1t}) + (1 - f(e_{igt}, e_{jgt}))(\tau_{i0t} + \tau_{j0t})$$

Government Budget Constraint (11)

(9)

$$f(e_{igt}, e_{jgt}) (\tau_{i1t} + \tau_{j1t}) + (1 - f(e_{igt}, e_{jgt})) (\tau_{i0t} + \tau_{j0t}) \leq A(Z_{at-1})(e_{iat} + e_{jat}) - f(e_{igt}, e_{jgt})k_L$$

Feasibility Constraint (12)

Due to the risk neutrality of the utility functions of agents, the concavity of productivity functions $A(Z_a)$, $\Gamma(Z_g)$ and the Inada condition, the optimal taxation and investment plans take the form of a stopping-time problem: at the beginning of the economy, when the marginal productivity of investment is higher than the marginal value of consumption, the institutional agent taxes all income from the other agents to fund investments in productivity. At some threshold value of productivity, taxation drops and no further investments are made. Non-taxable production is consumed thereafter.

However, trade between agents and inducing high effort levels for one of the types (or both) require specific threshold level of production of the non-specialized good, which are endogenously determined by investment plan z_a . Specifically, trade can take place between *i* and *j* only if the non-specialized production of agent *i* is sufficient to cover the cost of production in the low-cost state:

$$A(Z_{at-1})e_{iat} \geqslant k_L \tag{13}$$

Inducing high effort for one type of agent requires that the aggregate production of the non-specialized good minus the expected production costs of the specialized good are greater than the expected reduction in taxation:

$$A(Z_{at-1})\left(e_{iat} + e_{jat}\right) - f(\overline{e}, e_{\zeta})\Gamma(Z_{gt-1})k_L \ge f(\overline{e}, e_{\zeta})\left(-\frac{\Gamma(Z_{gt-1})\left[\beta^{min}\left(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})\left(v - k_L\right) - (\overline{c} - \underline{c})\right]\right)}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})}\right)$$
(14)

Similarly, incentivizing both agents to exert high effort level requires:

$$A(Z_{at-1})\left(e_{iat} + e_{jat}\right) - f(\overline{e}, \overline{e})\Gamma(Z_{gt-1})k_L \ge f(\overline{e}, \overline{e})\left(-\frac{\Gamma(Z_{gt-1})\left[\beta^{max}\left(f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})\right)\left(v - k_L\right) - (\overline{c} - \underline{c})\right]}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})}\right) + f(\underline{e}, \overline{e})\left(-\frac{\Gamma(Z_{gt-1})\left[\beta^{min}\left(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})\right)\left(v - k_L\right) - (\overline{c} - \underline{c})\right]}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})}\right)$$
(15)

In inequalities (14),(15), $\beta^{min} = \min\{\beta_i, \beta_j\}$ and $\beta^{max} = \max\{\beta_i, \beta_j\}$. It is easy to solve the problem when these constraints are not binding, which is the case when the initial value of productivity for the non-specialized good is sufficiently high. In this case the optimal social plan induces both agents to exert high effort level by reducing taxation in the low-cost state and increasing it in the high-cost state (which are truth-fully revealed through trade). It also increases capital stocks in period zero up to the point where the future marginal increase in production is equal to the marginal cost of production for both goods. Formally, this solution is defined by the equations below:

$$\tau_{\xi 0t} - \tau_{\xi 1t} = -\frac{\Gamma(Z_{gt-1}) \left[\beta_{\xi} \left(f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})\right) \left(v - k_{L}\right) - (\overline{c} - \underline{c})\right]}{f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})} \quad \text{for } \xi \in \{i, j\}$$

$$z_{a0}$$
 such that: $\sum_{t=1}^{T} \delta^t \frac{\partial A(Z_{a0})}{\partial Z_a} \left(e_{ia}^*(Z_{a0}) + e_{ia}^*(Z_{a0}) \right) = 1$, where $Z_{a0} = z_{a0}$

$$z_{g0}$$
 such that: $\sum_{t=1}^{T} \delta^t \frac{\partial \Gamma(Z_{g0})}{\partial Z_g} \left(f(\overline{e}, \overline{e})(v - k_L) - 2\overline{c} \right) = 1$, where $Z_{g0} = z_{g0}$

The above equations characterize the problem only for very high values of A(0). If A(0)

is such that any of the feasibility constraints (13)-(15) is not satisfied at time t = 0, then the optimal investment plan z_a depends on the marginal benefits and costs of prolonging investment in future periods. The benefits come from the increased future production of the autarchic good and satisfying the minimum production level required for trade to take place or for providing subsidies¹⁸. The costs come from the value of foregone consumption in the period of investment.

Due to the fact that equations (13) to (15) define threshold values, which are determined endogenously by past investment and taxation, there are multiple different cases to consider regarding optimal investment and taxation plans. Diagrams 1 to 3 in Appendix II provide all possible cases, but in the following subsection we examine only the most interesting of these cases. Before doing so, we define some useful threshold values, which will be used in the analysis thereafter. The following threshold values $\underline{K}(Z_{gt-1})$ and $\overline{K}(Z_{gt-1})$ represent the aggregate production of the autarchic good required for the subsidization of one or two types of agents respectively form equations (14) and (15).

Let
$$\underline{K}(Z_{gt-1}) = f(\overline{e}, \underline{e}) \left(-\frac{\Gamma(Z_{gt-1}) \left[\beta^{min}(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e}))(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})} \right) + f(\overline{e}, \underline{e}) k_L$$

Let $\overline{K}(Z_{gt-1}) = f(\overline{e}, \underline{e}) \left(-\frac{\Gamma(Z_{gt-1}) \left[\beta^{min}(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e}))(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})} \right)$
 $+ f(\overline{e}, \overline{e}) \left(-\frac{\Gamma(Z_{gt-1}) \left[\beta^{max}(f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e}))(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) + f(\overline{e}, \underline{e}) k_L$

As before, $\beta^{min} = \min\{\beta_i, \beta_j\}$, $\beta^{max} = \max\{\beta_i, \beta_j\}$. Define by $\widetilde{Z}_a, \underline{Z}_a, \overline{Z}_a$ the minimum require physical capital of type a, such that the inequalities (13) to (15) hold respectively at time zero. Formally:

$$\widetilde{Z}_{a} : A(\widetilde{Z}_{a})e_{ia}^{*}(\widetilde{Z}_{a}) = k_{L}$$

$$\underline{Z}_{a} : A(\underline{Z}_{a})\left(e_{ia}^{*}(\underline{Z}_{a}) + e_{ja}^{*}(\underline{Z}_{a})\right) = \underline{K}(0)$$

$$\overline{Z}_{a} : A(\overline{Z}_{a})\left(e_{ia}^{*}(\overline{Z}_{a}) + e_{ja}^{*}(\overline{Z}_{a})\right) = \overline{K}(0)$$

¹⁸Which generate extra value by increasing the volume of trade of the specialized good.

If the solutions to the above equations do not exist (and since effort exertion is an increasing function of productivity), there are two possible cases to consider for each equation. Either the required value of Z_a is so low that it violates the non-negativity constraint for the capital stock or the limit $\lim_{Z_a\to\infty} A(Z_a)(e_{iat} + e_{jat})$ is lower than the required threshold.

The first case is the case where A(0) is sufficiently high so that feasibility constraints for trade and effort exertion are not binding and we go back to the analysis of the unconstrained problem provided above. In the second case, the respective feasibility constraint is always binding, which implies that either trade will never take place, irrespectively of the capital stock of the economy, or inducing high effort for at least one or both agents is not attainable. However, the second class of results can also arise in the case where the required capital stock for trade (or inducing high effort) is attainable, but the cost of foregone consumption is too high and such an investment plan is not optimal. Therefore, in terms of economic consequences, we lose nothing by restricting our attention to the cases where the critical values \tilde{Z}_a , \underline{Z}_a and \overline{Z}_a exist and are non-negative.

The importance of these thresholds is that we can examine the optimal investment and taxation plans when one of these constraints is more difficult to satisfy than the others. Notice that because $\underline{K}(Z_{gt})$ is always greater than $\overline{K}(Z_{gt})$, then $\overline{Z}_a > \underline{Z}_a$, which means that feasibility constraint (15) is always more difficult to satisfy than (14). Furthermore, for each one of these critical values and given some plan z_a , which utilizes all non-specialized production in each period for investments in productivity and subsidizing effort levels, there exists a point t in time such that the accumulated capital stock Z_a reaches the respective critical value. Define \tilde{t}, \underline{t} and \overline{t} as the respective points in time and assume that each one of them is less than T. Under these assumption and results we examine the following case of interest.

4.4 An Economy with non-Monotonic Regulation

We now analyze the case where the three feasibility constraints (13)-(15) are binding at time zero. We derive the required conditions on parameter values so that the economy passes through the different stages of economic development (no-trade, trade with low effort levels, trade with high effort levels). We also find the conditions for regulation to change non-monotonically over these stages. Enforcement institutions are required at the point where productivity in good a is high enough to support trade, but contract law becomes relevant only when subsidization of trade for at least one type is feasible. In turn, the increase in the trade of good g, generated by regulation, spurs further economic growth by increasing its marginal value and this leads to further investments in the productivity of this good. Therefore, we claim that the interaction between markets and institutions goes in both directions¹⁹.

As before, there are different combinations of assumptions that can give similar qualitative results in our model²⁰. We examine the conditions that present the most detailed interpretation of economic development and enforcement institutions evolution. The main requirement is that the continuation value of investment in productivity of goods *a* and *g* is greater than the cost of foregone consumption in all three threshold points $(\tilde{t}, \underline{t}, \overline{t})$. This generates optimal investment paths for five different stages of development: $[0, \tilde{t}], [\tilde{t}, \underline{t}], [\underline{t}, \overline{t}], [\overline{t}, t^*]$ and $[t^*, T]$. These are described below:

Stage 1: $[0, \tilde{t}]$

Assume that $\widetilde{Z}_a < \overline{Z}_a$. This implies that the most difficult to satisfy constraint is the subsidization of both types of agents. Define \widetilde{t} as the time period which satisfies the following condition and growth path: $\sum_{t=0}^{\widetilde{t}} z_{at} = \widetilde{Z}$, $z_{at} = A(Z_{t-1})(e_{iat-1}^* + e_{jat-1}^*)$ and assume that $\frac{\partial A(\widetilde{Z}_a)}{\partial \widetilde{Z}_a}[e_{ia}^*(\widetilde{Z}_a) + e_{ja}^*(\widetilde{Z}_a)]\left(\sum_{t=\widetilde{t}}^T \delta^{t-\widetilde{t}}\right) > 1$. The last assumption means that the net marginal benefit of investment z_a at the critical value \widetilde{Z}_a is positive, so that investment in the productivity of good a must continue beyond this threshold. Therefore, z_{at} follows the state-independent growth path $z_{at} = A(Z_{t-1})(e_{iat-1}^* + e_{jat-1}^*)$ up to time \widetilde{t} . Call this investment plan $IP_{\widetilde{t}}^*$. During stage 1, productivity for good a is low and the autarchic production of type i is not sufficient for covering the cost of production of good g in any state. Trade does not take place, but production of a is taxed away and invested in increasing Z_a .

Stage 2: $[\tilde{t}, \underline{t}]$ Define \underline{t} such that: $\widetilde{Z}_a + \sum_{t=\tilde{t}+1}^{\underline{t}} z_{at} = \underline{Z}_a$

 z_{at} , z_{gt} are defined by the following investment paths:

 $z_{gt} = A(Z_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \Gamma(Z_{gt})f(\underline{e}, \underline{e})k_L , \ Z_{gt} = Z_{gt-1} + z_{gt} \text{ and } z_{at} = 0$

²⁰See Appendix II.

¹⁹The paper by Dhillon and Rigolini is also important in that respect, as the determination of prices and the use of formal and informal channels of enforcement arise endogenously. Our main focus is on the impact of the design of contract law.

$$\text{if} \quad \frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) < \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right)$$

otherwise, $z_{at} > 0$ and z_{at} , z_{gt} are such the marginal returns are equalized:

$$\frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) = \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right) + z_{at} + z_{gt} = A(Z_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \Gamma(Z_{gt-1})f(\underline{e}, \underline{e})k_L$$

Call the optimal investment path above $IP_{\underline{t}}^*$. At time \tilde{t} there is enough production of good a by agents of type i, so that trade arises in the economy, but, since $\underline{Z}_a > \widetilde{Z}_a$, there is not enough production for subsidizing effort exertion. This constraint holds until the physical capital reaches the critical value \underline{Z}_a , which happens in period \underline{t} . Also at the same time (\tilde{t}) enforcement institutions emerge but no restrictions on enforceability are required.

On the other hand, after \tilde{t} , due to the Inada conditions, investment takes place in productivity Γ and the optimal investment plan must divide available production between z_a and z_b . The optimal rule is to invest production to the productivity of good g only, until the marginal returns of production are equalized between goods aand g. Thereafter, investment is divided between the two goods so as to maintain the equality of marginal returns. This is represented by the first (the "if") and second (the "otherwise") part respectively of IP_t^* .

Stage 3: $[\underline{t}, \overline{t}]$ The main logic and intuition proceeds in the same way as in stage 2. Define \overline{t} such that: $\underline{Z}_a + \sum_{t=\underline{t}+1}^{\overline{t}} z_{at} = \overline{Z}_a$. Furthermore assume that: $\frac{\partial A(\underline{Z}_a)}{\partial Z_a} \left[e_{ia}^*(\underline{Z}_a) + e_{ja}^*(\underline{Z}_a) \right] \left(\sum_{s=\underline{t}}^T \delta^{s-\underline{t}} \right) > 1$ $\frac{\partial \Gamma(\underline{Z}_{gt})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=\underline{t}}^T \delta^{s-\underline{t}} \right) > 1$ $\sum_{t=\underline{t}}^T \delta^t \left[A(Z_{at-1}) \left(e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right) + \Gamma(Z_{gt-1}) \left(f(\overline{e}, \underline{e})v - \overline{c} - \underline{c} \right) \right] > \sum_{t=\underline{t}}^T \delta^t \left[A(\hat{Z}_{at-1}) \left(\hat{e}_{ia}^*(\hat{Z}_{at-1}) + \hat{e}_{ja}^*(\hat{Z}_{at-1}) \right) + \Gamma(\hat{Z}_{gt-1}) \left(f(\underline{e}, \underline{e})v - 2\underline{c} \right) \right]$

The first two conditions ensure that it is optimal to invest both in the productivity of good a and g beyond time period \underline{t} , at which point there is enough production of good

a so that one of the two agents can be subsidized to exert high effort. Notice that the marginal benefit of one of the agents exerting high effort is the same irrespectively of who is subsidized, due to the symmetry of the probability function in terms of effort levels. However, the subsidy required for inducing high effort is lower for the agent with the lower bargaining power. This is the reason why we have defined \underline{Z}_a to depend on β^{min} . Let us assume that type ξ is the one with the low bargaining power.

At time \underline{t} , it is optimal for type ξ to receive subsidy conditional on trade, if the overall social surplus from the increased probability of trade and the slower increase in productivity is greater than the social surplus under the lower probability of trade but the faster increase in productivity. This trade-off between the probability of trade and productivity growth comes from the fact that both must be funded from the taxation imposed on agents and they both face ultimately the same feasibility constraint. The third condition ensures that paying out subsidies to one type of agents Pareto dominates not paying subsidies at all. The investment path with partial subsidies $\{z_a, z_g\}$ and without any subsidies $\{\hat{z}_a, \hat{z}_g\}$, are given below:

$$\begin{aligned} z_{gt} &= A(Z_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \underline{K}(Z_{gt}) \quad , \quad Z_{gt} = Z_{gt-1} + z_{gt}, \quad z_{at} = 0 \ , \\ \text{if} \quad \frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) < \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right) \end{aligned}$$

otherwise, $z_{at} > 0$ and z_{at} , z_{gt} are such the marginal returns are equalized: $\frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) = \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\overline{e}, \underline{e})(v - k_L) - \underline{c} - \overline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right),$ $z_{at} + z_{gt} = A(Z_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \underline{K}(Z_{gt-1})$

$$\begin{aligned} \hat{z}_{gt} &= A(\hat{Z}_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \Gamma(\hat{Z}_{gt})f(\underline{e},\underline{e})k_L \quad , \quad \hat{Z}_{gt} = \hat{Z}_{gt-1} + \hat{z}_{gt}, \ \hat{z}_{at} = 0 \\ \text{if} \quad \frac{\partial A(\hat{Z}_{at-1})}{\partial Z_a} \left[e^*_{ia}(\hat{Z}_{at-1}) + e^*_{ja}(\hat{Z}_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) < \frac{\partial \Gamma(\hat{Z}_{gt-1})}{\partial Z_g} \left[f(\underline{e},\underline{e})(v-k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right) \end{aligned}$$

otherwise, $\hat{z}_{at} > 0$ and \hat{z}_{at} , \hat{z}_{gt} are such the marginal returns are equalized: $\frac{\partial A(\hat{Z}_{at-1})}{\partial Z_a} \left[e^*_{ia}(\hat{Z}_{at-1}) + e^*_{ja}(\hat{Z}_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) = \frac{\partial \Gamma(\hat{Z}_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right)$ $\hat{z}_{at} + \hat{z}_{gt} = A(\hat{Z}_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \Gamma(\hat{Z}_{gt-1})f(\underline{e}, \underline{e})k_L$

Therefore, the optimal investment plan for $[\underline{t}, \overline{t}]$ (call it $IP_{\overline{t}}^*$) is characterized by the three inequalities and the investment plan $\{z_a, z_g\}$ above²¹.

²¹See also Appendix II for the possible cases when some of the above inequalities are violated.

Stage 4: $[\bar{t}, t^*]$ Similarly to stage 3, define t^* such that:

$$\frac{\partial A(Z_a)}{\partial Z_a} \left[e_{iat}^*(Z_a) + e_{jat}^*(Z_a) \right] \left(\sum_{s=t^*}^T \delta^{s-t^*} \right) = \frac{\partial \Gamma(Z_{gt^*})}{\partial Z_g} \left[f(\overline{e}, \overline{e})(v - k_L) - 2\overline{c} \right] \left(\sum_{s=t^*}^T \delta^{s-t^*} \right) = 1$$

Also assume that:

$$\begin{aligned} \frac{\partial A(\overline{Z}_{a})}{\partial Z_{a}} \left[e_{ia}^{*}(\overline{Z}_{a}) + e_{ja}^{*}(\overline{Z}_{a}) \right] \left(\sum_{s=\overline{t}}^{T} \delta^{s-\overline{t}} \right) > 1 \\ \frac{\partial \Gamma(Z_{g\overline{t}})}{\partial Z_{g}} \left[f(\overline{e}, \underline{e})(v - k_{L}) - \overline{c} - \underline{c} \right] \left(\sum_{s=\overline{t}}^{T} \delta^{s-\overline{t}} \right) > 1 \\ \sum_{t=\overline{t}}^{T} \delta^{t} \left[A(\check{Z}_{at-1}) \left(e_{ia}^{*}(\check{Z}_{at-1}) + e_{ja}^{*}(\check{Z}_{at-1}) \right) + \Gamma(\check{Z}_{gt-1}) \left(f(\overline{e}, \overline{e})v - 2\overline{c} \right) \right] > \\ \sum_{t=\overline{t}}^{T} \delta^{t} \left[A(Z_{at-1}) \left(e_{ia}^{*}(Z_{at-1}) + e_{ja}^{*}(Z_{at-1}) \right) + \Gamma(Z_{gt-1}) \left(f(\overline{e}, \underline{e})v - \overline{c} - \underline{c} \right) \right] \end{aligned}$$

These conditions are equivalent to the conditions specified for periods $[\underline{t}, \overline{t}]$. The first two ensure that investment in productivity of good a and g are optimal after $t = \overline{t}$. At this point of time, there is adequate production of the autarchic good so that subsidizing the effort levels of both types is feasible. As before, subsidizing both agents is optimal if it generates greater overall utility than subsidizing only one agent, which, given the conditions we have specified thus far, is better than no subsidization for any type. The required assumption for this result is the third condition. Call the optimal investment plan until t^* as $IP_{t^*}^*$. $IP_{t^*}^*$ incorporates all the previous optimal investment plans up to \overline{t} plus the investment plan $\{\tilde{z}_{at}, \tilde{z}_{gt}\}$ thereafter. This is given below:

$$\begin{split} \check{z}_{gt} &= A(\check{Z}_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \overline{K}(\check{Z}_{gt}), \ \check{Z}_{gt} = \check{Z}_{gt-1} + \check{z}_{gt}, \ \check{z}_{at} = 0\\ \text{if} \quad \frac{\partial A(\check{Z}_{at-1})}{\partial Z_a} \left[e^*_{ia}(\check{Z}_{at-1}) + e^*_{ja}(\check{Z}_{at-1}) \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right) < \frac{\partial \Gamma(\check{Z}_{gt-1})}{\partial Z_g} \left[f(\bar{e},\bar{e})(v-k_L) - 2\bar{c} \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right) \end{split}$$

otherwise, $\check{z}_{at} > 0$ and \check{z}_{at} , \check{z}_{gt} are such marginal returns are equalized: $\frac{\partial A(\check{Z}_{at-1})}{\partial Z_a} \left[e^*_{iat}(\check{Z}_{at-1}) + e^*_{jat}(\check{Z}_{at}) \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right) = \frac{\partial \Gamma(\check{Z}_{gt-1})}{\partial Z_g} \left[f(\bar{e},\bar{e})(v-k_L) - 2\bar{e} \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right),$ $\check{z}_{at} + \check{z}_{gt} = A(\check{Z}_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \overline{K}(\check{Z}_{gt-1})$

The plan $\{z_{at}, z_{gt}\}$ follows the same growth path and is subject to the same feasibility constraints as the optimal plan for the periods $[\underline{t}, \overline{t}]$. Of course, by the third inequality

above, it is suboptimal to $\{\check{z}_{at}, \check{z}_{gt}\}$, and is not part of the equilibrium path (See also Appendix II for the cases where the inequalities above do not hold).

Stage 5: $[t^*, T]$

After t^* investment falls to zero, since the marginal utility increase by additional investment in either types of physical capital is lower than the marginal cost of foregone consumption. In this case, agents receive subsidies whenever they trade, pay taxes whenever they do not trade and consume the remainder of the production. The evolution of regulation depends on the investment path for Z_g and the optimally induced effort levels. Recall that the maximum permitted transfers are given by $\overline{p} = \Gamma(Z_{gt}) \left(k_H + \beta_j (v - k_L) - \frac{\overline{c-c}}{f(e_i,\overline{e}) - f(e_i,\underline{e})} \right).$

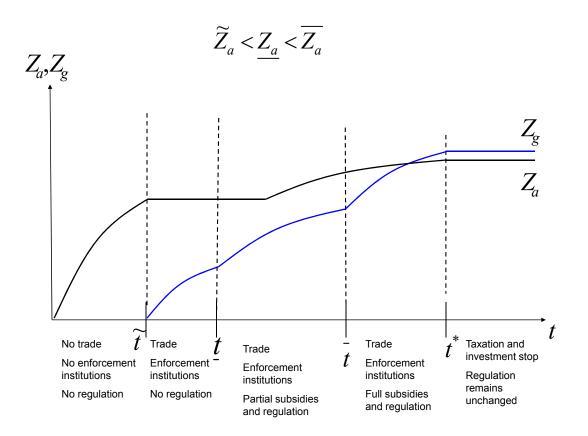


Figure 3: Capital-Accumulation Paths

Figures 3 and 4 give the graphical representation of the economy according to the

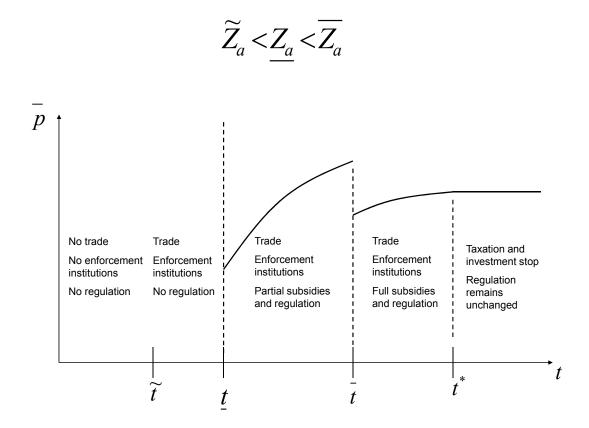


Figure 4: Maximum enforceable transfers

conditions above. A first note is that they are drawn as if time is continuous. While our model is one of discrete time, the figures can be closely approximated if T takes sufficiently high values. Figure 3 depicts the equilibrium path of capital stocks Z_a and Z_g and Figure 4 depicts the equilibrium path of contract law, as represented by the maximum enforceable transfer \overline{p} . These graphs also present the different stages of economic development.

To recapitulate, during $[0, \tilde{t}]$ productivity Z_a is very low and trade is infeasible. There is no need for enforcement institutions or regulation and taxation is used in increasing Z_a . This stage reflects a rather primitive stage of economic organization.

In stage $[\tilde{t}+1, \underline{t}]$ trade becomes feasible and enforcement institutions are required to support it by making private agreements enforceable. However, providing tax-breaks to agents in order to incentivize high effort remains infeasible and hence regulation is still not required. Any agreement on exchanging specialized goods is enforceable. On the other hand, public spending can be used for increasing both types of capital stock, but, since the marginal increase in productivity of g is much higher than the marginal productivity of good a, taxation flows only to the former. This continues until effective marginal returns are equalized, which depends on parameter values.

In periods $[\underline{t} + 1, \overline{t}]$, Z_a exceeds the threshold value \underline{Z}_a and partial (one-type) subsidization of trade takes place. At this point of development, limitations on the set of enforceable transfers are set in order to make tax-breaks effective in inducing high effort. Thus the probability of trade increases for each group. On average, trade and its marginal value increase as well, which induces greater investment in Z_g . This is shown in the diagram by the kink in the slope of Z_g at period \underline{t} . In our model, higher investment in Z_g also means that the optimal stopping time for investment t^* is delayed. Moreover, as Γ_t increases, \overline{p} also increases, which means that regulation is initially severe but it is subsequently relaxed. This is an important stage of the development process, where one can see the interaction between contract law, specialization and productivity growth and could be loosely interpreted as a stage akin to merchantilism before industrialization²².

The next stage, $[\bar{t}+1, t^*]$, is the stage where the economy develops fully its productive capacity. Investments in Z_a and Z_g reach their peak in t^* and tax-breaks are given to both types. The feedback effect between contract law and productivity is repeated. Furthermore, as Figure 4 depicts, if $f(\underline{e}, \overline{e}) - f(\underline{e}, \underline{e}) > f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})$, then contract law tightens again (\overline{p} decreases). This gives rise to a non-monotonic pattern of regulation over time, which may explain why it is difficult in empirical studies to find a robust effect between contractual institutions and growth.

Finally, the remaining periods of the economy, $[t^*+1, T]$, represent a fully developed economy. Capital stocks and trade have reached their optimum level and taxation is used solely for inducing high effort levels. The remainder of production is used for consumption purposes.

 $^{^{22}}$ It is also noteworthy to mention that Landes (1998) provides evidence that, despite common beliefs, the medieval period experienced a rapid increase in the productivity of the agricultural sector. This is in line with the argument made here that the initial growth in productivity of the non-specialized sector precedes specialization and the period of severe regulation.

4.5 The non-monotonicity between enforcement costs and development

As we mentioned in the introduction, one of the purposes of the paper is to show that enforcement costs may not be monotonically related to economic growth and, therefore, the empirical specification by Acemoglu and Johnson (2005) may not be the appropriate one for testing the relationship between contract law and development. As we have shown in the previous subsection, it is possible that the evolution of regulation is non-monotonic over the process of economic development. That is, there may be periods where regulation is relaxed and the contractual space expands, while other periods where the contractual space shrinks (as shown in Figure 4).

If this is the case, and if one makes the additional (and plausible) assumption, that contract enforcement costs are positively correlated with the set of enforceable agreements, then it follows that contract enforcement costs may not be positively correlated with growth. Of course, in our model, for theoretical reasons, we have implicitly assumed that contract enforcement costs are zero. This helped us to contrast our results, where the limits on contractual space arise endogenously, with the results of the previous literature, where, due to exogenous enforcement costs, the contractual space is incomplete. But we can modify our model to include strictly positive enforcement costs.

Consider the following modification of our model. Assume that in the economy there are two categories of ploughs. Those which are described by our model and which suffer from a hold-up problem in their production, and another category, which is unaffected by any hold-up considerations. However, the second variety of ploughs (call them the class-two ploughs), varies in terms of its value for the farmer, while it has a constant cost k_2 . Say that, for class-two, v is distributed uniformly between $[0, \bar{v}]$, with $\bar{v} > \max{\{\bar{p}_t\}}$. Furthermore, a plough of class-two may turn out to be defective after its sale, in which case the farmer can ask for a refund, if he proves his case in the court. This is necessary, if his output in terms of wheat can be verified by the court but not the seller (as is usually the case).

In this economy, as long as the class-two category is not too large a fraction of trade (so that the ex-post inefficiencies generated by banning some of these exchanges are not too large), it is still optimal to not allow some contracts from being written in equilibrium (for optimal incentives in the production of class-one ploughs). Moreover, the dynamic evolution of contract law is still represented by Figure 4 (under the relevant conditions) and the aggregate transactions for ploughs of class-two are positively correlated with \overline{p}_t . As a consequence, the number of cases brought to courts and aggregate

enforcement costs are positively correlated with the design of contract law (\bar{p}_t) as well. Of course, the plausibility of the result depends on the credibility of the assumptions.

But still the assumption (the positive correlation between contract-enforcement costs and the set of enforceable agreements) can be justified in terms of intuition. This is because one expects that the greater the aggregate number of transactions in an economy, the greater the number of cases that end up in courts and hence the higher the aggregate enforcement costs (and the costs per case, due to the dis-economies of scale exhibited by bureaucracies). And the aggregate number of transactions is expected to be positively correlated with the size of the set of permissible transactions. Of course, these hypotheses, though plausible, remain to be tested empirically as well.

4.6 Discussion

As we mentioned before, the institutional agent stands for a governor or a monarch, who is bound by the agreement she has made with the rest of the citizens in period zero. She agent plays a double role in this economy. The standard economic role implied by most models of public economics is the role of the government, which collects taxes and allocates public expenditures in economic activities where the market mechanism fails to provide efficient outcomes. In our model, investment in the production technology is a public good, due to the assumption of the continuum of agents and the assumption that productivity parameters are economy-wide.

The second role of the institutional agent is her role as an enforcement authority for private agreements. As we have shown, in the setup of this paper, it may not always be optimal for the enforcement authority to guarantee the conduct of trade, unlike the previous literature. Because agents are perfectly rational, they will never honor their part of agreement when they know that it will not be enforced on them. Hence making some private agreements non-enforceable is equivalent to effectively forbidding them. So one can interpret the endogenously determined set of non-enforceable private agreements as laws of banning some types of transactions and, as we have shown, this can have a positive effect on social welfare.

Overall, the analysis of this subsection presents an economy which moves through all the stages of development. It starts from a point where production is limited to activities who do not suffer from hold-up problems, it moves to a period where trade begins and enforcement authorities are necessary and finally it ends up in a stage with limitations on enforceable agreements (regulation). Furthermore, the resources spent for incentivizing agents to exert high effort is proportional to the productivity parameter Γ_t and hence as the value of trade grows so do the optimal subsidies. This can be interpreted as an endogenous growth of enforcement institutions, at least in terms of the resources devoted to their cause. Moreover, the exertion of high effort increases the marginal value of trade and, as a consequence, it increases the threshold value for the capital stock Z_g . This implies that the overall investments made in the productivity of trade are higher and this generates a positive feedback loop from economic growth to contract law and back to economic growth. In other words, our model provides a formalization of the argument that growth and law are inexorably entangled and the development of one has consequences for the other.

5 Conclusion

The main purpose of the paper is to relate the process of economic development with the emergence and evolution of enforcement institutions and contract law. We demonstrate the importance of restrictions on enforceable agreements for increasing social welfare and growth. In equilibrium, the relationship between economic growth and the intensity of regulation may be non-monotonic and this may explain why empirical studies do not find a positive correlation between enforcement institutions and development. Finally, our model derives the design of enforcement institutions and the centralization of authority as equilibria phenomena.

We believe that our model can be extended in various directions in order to explain different phenomena related to the process of economic development, institutional design and regulation. Notice that in the models of section 2 and 3, there is no loss of efficiency. Despite the fact that regulation is necessary, all socially valuable transactions take place in equilibrium. So regulation does not impose any cost from an efficiency point view, which goes against standard economic theory and intuition. However, we can extend the model to incorporate multiple sources of trade among agents. In this case, regulation generates social value by underpinning the mechanism, which induces optimal incentives, but destroys social value by forbidding some welfare enhancing transactions. The trade-off between incentives and economic freedom is even more clear in the extended model. We believe that we can use this more elaborate model of the design of optimal regulation in order to explain the differences in regulatory regimes that we observe between economies.

Furthermore, our model can be modified in order to generate a theory of regulation cycles. In the current form of the model, the set of enforceable agreements changes whenever the number of agents, who need to be incentivized to exert high effort, changes. But in a model with a stream of new traded good being introduced at different stages of economic development, the introduction of a new good is associated with a tightening of regulatory restrictions, which are subsequently relaxed. This cycle is then repeated every time a new good is introduced. Thus, we can have a model of regulation cycles which are related to the process of innovation and technological advance.

Similarly, the model can be used to relate the role of enforcement institutions with the degree of specialization in the economy and the organizational complexity by assuming that new production processes require the cooperation of a larger set of individuals who face a multi-agent version of the hold-up problem. This model is able to generate regulation cycles and relate the degree of specialization to the existence of appropriate institutional restrictions at the same time.

Another assumption that we can relax is the assumption of the mutual exclusivity between productive and governmental activities. By allowing institutional agents to have a certain span of control, which they have to allocate between their institutional and their economic role, we can provide a more rigorous analysis of how and under what conditions centralization of power emerges. This can provide us with a more complete theory of institutional size and how it may vary along the development path of an economy.

For all the above reasons, we believe that our model is an interesting contribution to the literature, which highlights the importance of enforcement institutions to the process of development, and we believe that this research area is a fruitful ground for future work.

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Appendix I

Proposition 1: Let $\overline{p} = k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})}$. Consider the social contract S^* , which defines:

i) if trade of good g takes place, agent ξ receives subsidy (negative taxation) $\tau_{\xi 1} = (1 - f(\overline{e}, \overline{e})) \left(\beta_{\xi}(v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) < 0.$ ii) if trade of good g does not takes place, agent ξ pays out taxation $\tau_{\xi 0} = f(\overline{e}, \overline{e}) \left(-\beta_{\xi}(v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) > 0.$ iii) any private contract $\pi(0, \hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is nonenforceable.

Then S^* implements the first best effort levels and it is renegotiation-proof.

Proof of Proposition 1: Without any subsidization of effort, the farmer and the producer will trade the good only in the low-cost state at the expected price $p = (1 - \beta_i)v - \beta_i k_L$. The expected utility from trade is $v - p = \beta_i (v - k_L)$ for the farmer and $p - k_L = (1 - \beta_i)(v - k_L)$ for the plough-maker.

Agent ξ exerts high effort in the production of good g if the expected utility of high effort is higher than the expected utility of low effort. Assuming that trade takes place at the expected price and given the beliefs of ξ for the effort level of ζ :

$$e_{\xi} = \overline{e} \Leftrightarrow f(\overline{e}, e_{\zeta}) \left(\beta_{\xi}(v - k_L) - \tau_{\xi 1}\right) + \left(1 - f(\overline{e}, e_{\zeta})\right) \left(-\tau_{\xi 0}\right) - \overline{c} \ge f(\underline{e}, e_{\zeta}) \left(\beta_{\xi}(v - k_L) - \tau_{\xi 1}\right) + \left(1 - f(\underline{e}, e_{\zeta})\right) \left(-\tau_{\xi 0}\right) - \underline{c} \Leftrightarrow$$

$$\tau_{\xi 0} - \tau_{\xi 1} \ge -\beta_{\xi} (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})}$$

Since we require both agents to exert high effort²³: $\tau_{\xi 0} - \tau_{\xi 1} \ge -\beta_{\xi}(v - k_L) + \frac{\overline{c} - c}{f(\overline{c},\overline{c}) - f(\underline{c},\overline{c})}$. This expression gives the minimum required wedge between taxation in the two states in order to induce high effort for ξ . Notice that the right hand side of the inequality is positive due to the condition imposed by inequality (1) in section 2. On the other hand, incentive compatibility requires that the producer of the good, who suffers from the high production costs in state H, does not want to trade in that state. The farmer may be willing to offer him an increased price in order to cover the high production cost, in which case incentive compatibility is impossible to satisfy without restrictions

²³One may worry that coordination failures are possible if both agents believe that the other agent will exert low effort level and if $f(\overline{e},\overline{e}) - f(\underline{e},\overline{e}) > f(\overline{e},\underline{e}) - f(\underline{e},\underline{e})$. In this case we can define the difference between taxation levels in terms of the $min\{f(\overline{e},\overline{e}) - f(\underline{e},\overline{e}), f(\overline{e},\underline{e}) - f(\underline{e},\underline{e})\}$. This ensures that it is a strictly dominant strategy for each agent to exert high effort.

on the transfers that the two agents can do. Indeed, if the farmer pays an expected price \hat{p} , incentive compatibility for the producer requires (notice that in this case, since we are referring to the producer, $\xi = j$):

$$\hat{p} - k_H - \tau_{j1} \leqslant -\tau_{j0} \Leftrightarrow \tau_{j0} - \tau_{j1} \leqslant k_H - \hat{p}$$

Combining the two inequalities above, the combination of incentive compatibility and effort exertion conditions, gives:

$$-\beta_{j}(v-k_{L}) + \frac{\overline{c}-\underline{c}}{f(\overline{e},\overline{e})-f(\underline{e},\overline{e})} \leqslant k_{H} - \hat{p} \Leftrightarrow$$
$$\hat{p} \leqslant k_{H} + \beta_{j}(v-k_{L}) - \frac{\overline{c}-\underline{c}}{f(\overline{e},\overline{e}) - f(\underline{e},\overline{e})}$$
(16)

Any net transfer of resources higher than the right hand side of (16) will lead to violation of incentive compatibility and trade in the high cost state, which then leads to low-effort exertion. This is because agents know that they will trade and receive the subsidies in any state of the world. Let $\overline{p} = k_H + \beta_j (v - k_L) - \frac{\overline{c}-\underline{c}}{f(\overline{c},\overline{c}) - f(\underline{c},\overline{c})}$. First, note that \overline{p} is greater than the fair price p:

$$\overline{p} > p \Leftrightarrow k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})} > (1 - \beta_i)v + \beta_i k_L \Leftrightarrow k_H - k_L > \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$$

The inequality above holds, due to the assumptions of the model and inequality (1):

$$k_H - k_L > v - k_L > \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$$

Since $\overline{p} > p$, the producer will not accept to trade in the high cost state without receiving an additional payment above the "fair" price. So violation of incentive compatibility from one side requires a higher exchange price or a side contract, actions which themselves are verifiable. Second, the farmer is willing to pay a price as high, because the utility of trading under this price is greater than the utility of not-trading:

$$v - \overline{p} - \tau_{i1} > \tau_{i0} \Leftrightarrow v - k_H - \beta_j (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})} - (\tau_{i0} - \tau_{i1}) > 0 \Leftrightarrow$$

 $k_L - k_H + 2 \frac{\overline{c} - \underline{c}}{f(\overline{c}, \overline{c}) - f(\underline{e}, \overline{c})} > 0$, which holds due to inequality (3) in section 2.

Therefore, inducing high-effort exertion and incentive compatibility, require that

any transfer $\hat{p} > \overline{p}$ or any side-payment $\hat{q} > \overline{p} - p$ is non-enforceable. Required taxation conditional on trade is given by:

$$\tau_{\xi 0} - \tau_{\xi 1} \ge -\beta_{\xi} (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$$
(17)

$$f(\overline{e},\overline{e})\tau_{\xi 1} + (1 - f(\overline{e},\overline{e}))\tau_{\xi 0} = 0$$
(18)

This system provides the required taxation levels as given by the proposition. We turn now to the issue of renegotiation-proofness. Consider first the ex-post renegotiation. If the low-cost state arises then the farmer and the producer can enforce the trade and receive the subsidies by making the same trade agreement as the one they would have made in the absence of the social contract. This provides them the maximum expost payoff. Therefore, any renegotiation of the social contract can not increase their payoffs and agents have no incentive to renegotiate it with the institutional agent. If the high-cost state arises, then the two agents can increase their pay-offs if they modify the social contract so that private contracts are enforceable. In such a case, net transfers of resources between the two agents are allowed and they can extract the subsidies by trading at the pre-specified price p, even though this is suboptimal. Since it is the institutional agent who bears the true cost of this sub-optimal transaction, both i and jwould agree on it. However, the institutional agent anticipates the renegotiation of the social contract in the bad state. Furthermore, any credible reward the agents are willing to provide to him in order to agree to renegotiate does not cover for the losses by paying out the subsidies. Therefore, the institutional agent does not agree to renegotiate the social contract in period 4.

A similar type of argument shows that the institutional agent does not agree to renegotiate the social contract in the interim stage. \blacksquare

Proposition 2 below provides the necessary conditions for the implementation of first-best effort levels. It is proven for a more general environment, where effort levels are continuous and hence inequalities (1)-(2) are not necessary. Condition (3) is substituted for $v - k_L > k_H - v$, but the interpretation and role is the same. Due to the Revelation Principle, we restrict our attention to direct mechanisms S, where agents send messages about the state of the world and the mechanism determines if the good is traded or not, at what price and if subsidies are given.

Proposition 2: The existence of the institutional agent and the non-enforcement of

private contracts contingent on trade are necessary conditions for the implementation of first-best effort levels.

Proof of Proposition 2: First, we prove that any renegotiation-proof mechanism S that does not include an institutional agent ($\Sigma = \emptyset$) can not implement the first-best effort levels. Consider any direct mechanism $S(., \Sigma = \emptyset)$, and assume that the mechanism infers that the cost is low if the agents send the messages $\hat{\mathbf{m}}$. Implementation of the first-best effort levels requires that trade takes place and appropriate subsidies are provided only in the low-cost state. Therefore, in terms of ex-post utility, the combination of incentive compatibility and efficiency requires:

$$u_i(\hat{m}_i, \hat{m}_j | k_L) = v - k_L \geqslant u_i(m'_i, \hat{m}_j | k_L), \forall m'_i$$

$$\tag{19}$$

$$u_j(\hat{m}_j, \hat{m}_i | k_L) = v - k_L \geqslant u_j(m'_j, \hat{m}_i | k_L), \forall m'_j$$

$$\tag{20}$$

$$u_{\xi}(\widetilde{m}_{\xi}, \hat{m}_{\zeta}|k_H) = 0 \ge u_{\xi}(\hat{m}_{\xi}, \hat{m}_{\zeta}|k_H), \text{ for some } \widetilde{m}_{\xi} \neq \hat{m}_{\xi} \text{ and for some } \xi \in \{i, j\}$$
(21)

$$u_{\zeta}(m_{\zeta}, \widetilde{m}_{\xi}|k_H) = 0 , \forall m_{\zeta} and \zeta \neq \xi$$
(22)

Inequalities (19) and (20) imply that the aggregate subsidy given in the low-cost state is equal to the surplus: $\sum_{\xi} s_{\xi} = v - k_L$. Inequalities (21) and (22) require that at least one of the agents has the incentive not to trade if the high-cost state arises, while the other does not benefit from the non occurrence of trade.

If the mechanism $S(., \Sigma = \emptyset)$ allows for private contracts, contingent on messages (the equivalent of private contracts conditional on trade), (21) can not be satisfied. This is because $\sum_{\xi} s_{\xi} = v - k_L > k_H - v$ and therefore there is always a private contract $\pi(\hat{m}_i, q, \emptyset)$ such that $u_{\xi}(\hat{m}_{\xi}, \hat{m}_{\zeta} | k_H) + q > 0$ for both agents. Without loss of generality, assume that $S(., \Sigma = \emptyset)$ specifies a price p (not necessarily equal to $(1 - \beta_i)v + \beta_i k_L)$, if g is traded, and subsidies $\{s_i, s_j\}$, which satisfy (19) and (20). Then $u_i(\hat{m}_i, \hat{m}_j | k_H) = v - p + s_i$, $u_j(\hat{m}_j, \hat{m}_i | k_H) = p - k_H + s_j$ and $u_i(\hat{m}_i, \hat{m}_j | k_H) + u_j(\hat{m}_j, \hat{m}_i | k_H) = v - k_H + \sum_{\xi} s_{\xi} = (v - k_L) - (k_H - v) > 0$. Therefore, there exists a net-transfer q and a private contract $\pi(\hat{m}_{\xi}, p, \emptyset)$ such that $u_i(\hat{m}_i, \hat{m}_j | k_H) + q > 0$ and $u_j(\hat{m}_j, \hat{m}_i | k_H) - q > 0$, which violates the incentive compatibility-cum-efficiency condition.

If, on the other-hand, $S(., \Sigma = \emptyset)$ does not allow for the enforcement of private contracts, then there are two possibilities. Either inequality (21) can not be satisfied, in which case the result of the proposition holds, or (21) is satisfied. In this case, however, M is not ex-post renegotiation proof, because there exists another mechanism $S'(., \Sigma = \emptyset)$, which allows for private contract enforcement and makes both agents better off, as shown above. This completes the first part of the proposition, namely that the existence of the institutional agent is a necessary condition for the implementation of the first-best effort levels.

The necessity of the non-enforcement of the private contracts follows from the first part of the proof. Even if the mechanism specifies an institutional agent, inequalities (19)-(22) must still be satisfied, and with enforceable agreements of the form $\pi(\mathbf{m}, q, \emptyset)$, we already showed how (21) is violated.

Lemma 1: The optimal social contract S^* determines that the set Σ is of measure zero. That is, the total number of institutional agents is infinitesimal compared to the aggregate population.

Proof: Formally, $S^* = \arg \max_{S} \{m_{i(S)}U_i + m_{j(S)}U_j + m_{\Sigma S}r_{\sigma}^+\}$, where $m_{i(S)}$ is the mass of agents of type *i* who engage in productive activities according to *S*, $m_{j(S)}$ is the mass of type *j* agents, who engage in productive activities, and $m_{\Sigma(S)}$ is the mass of institutional agents set by S.

We show that any contract S, which assigns strictly positive mass to the set of institutional agents can not be socially optimal and will be not chosen. To see this, consider the social contract $S(\Sigma, \Phi(Q), \tau, z_a, z_g)$, with $m_{\Sigma} > 0$. Let S' be another social contract which specifies exactly the same taxation and investment schemes as $S(\tau' = \tau, z'_a = z_a, z'_g = z_g)$, has the same set of enforceable contracts $\Phi(Q)$, but specifies a smaller set of institutional agents Σ' , subset of $\Sigma(\Sigma' \subset \Sigma \Rightarrow m_{\Sigma'} < m_{\Sigma})$. S' also provides the same rewards **r** as S to all agent in the set Σ , even though some of the agents in this set are not institutional agents according to S'. In other words, S' keeps exactly the same vector of transfers of S but shrinks the set of institutional agents.

Then S' provides a Pareto improvement over S. This is because all agents who are occupied in productive activities under S continue to pay the same taxation and receive the same benefits from productivity investments in S', so their utility is unchanged. Also the institutional agents in the set Σ' are equally well-off as in S. But the agents in the set $\Sigma - \Sigma'$ are made better off because they receive the same reward as before, plus they are occupied in productive activities under S' and receive all the marginal benefits of their own production: S' is a Pareto improvement over S. Hence, any social contract with positive measure of institutional agents can not be part of the equilibrium strategies of the game. **Lemma 2:** The optimal social contract specifies the following rewards and punishments for the institutional agent: $\{r_{\sigma t}^+, r_{\sigma t}^-\} = \{(A(Z_{at})e_{\sigma a}^* - c_{ia0}(e_{\sigma a}^*)), \frac{1}{2}A(Z_{at})(e_{iat}^* + e_{jat}^*)\},$ with $e_{\xi at}^*$ being the optimal effort level for type ξ in period t.

Proof: Suppose that some social contract S is proposed in period t. The reward $r_{\sigma t}^+$ for the proposed institutional agent σ in period t must be at least as large as the discounted value of production of the good a that σ can produce in period t. Otherwise, she can vote against the proposal and receive her autarchic production. Therefore, in order to achieve the agreement of the institutional agent, the optimal social contract must set $r_{\sigma t}^+$ equal to $(A(Z_{at})e_{\sigma a}^* - c_{ia0}(e_{\sigma a}^*))$, where $e_{\sigma a}^*$ is such that $\frac{\partial c_{\sigma a0}}{\partial e_{\sigma a}} = A(Z_{at})$. On the other hand, the optimal social contract minimizes the resources allocated to non-productive activities, hence r_{σ}^+ can not be greater. These two conditions imply that $r_{\sigma t}^+ = \delta^t (A(Z_{at})e_{\sigma a}^* - c_{ia0}(e_{\sigma a}^*))$, where $e_{\sigma a}^*$: $\frac{\partial c_{\sigma a0}}{\partial e_{\sigma a}} = A(Z_{at})$.

 r_{σ}^{-} must be set sufficiently high, so as to prevent the institutional agent from not complying with the contract. Since the greatest amount of resources that the governor can extract from the economy is the aggregate production of the autarchic good, setting $r_{\sigma t}^{-}$ equal to $\frac{1}{2}A(Z_{at})(e_{iat}^{*}+e_{jat}^{*})$ is sufficient for making the governor to comply with her role in period t. The above mean that the rewards and punishments must increase with the increase in the productivity of the economy for the non-specialized good and the aggregate exerted effort level in its production. This generates the optimal punishment plan $\{\mathbf{r}_{\sigma}^{+}, \mathbf{r}_{\sigma}^{-}\}$, which is a vector with elements $r_{\sigma t}^{+}(A(Z_{at}))$ and $r_{\sigma t}^{-}(A(Z_{at}))$ for $t \in [0, T]$.

Proposition 3: Let $\overline{p} = \Gamma(Z_{gt-1}) \left(k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right)$. Incentive compatibility requires that any private contract $\pi(\hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is non-enforceable.

Proof: The proof follows from the proof of Proposition 1 in Appendix I. Incentive compatibility for type j requires that she prefers not to trade in the high cost state than to trade, which implies that $p + q - \Gamma_t k_H - \tau_{j1t} \leq -\tau_{j0t}$. At the same time, the difference in taxation conditional on trade must be sufficiently high so that agent j prefers to exert high effort. The condition is given by equation (6) in section 4.1. Combining the two conditions gives the condition that $p + q \leq \overline{p}$.

Appendix II

The figures in the following three pages present diagrammatically all the possible cases of investment plans. We do not derive the conditions under which these paths arise, but the required analysis is similar to section 4.4. The cases depend on the size of the three critical values \tilde{Z}_a , \underline{Z}_a and \overline{Z}_a . In each case, we describe the optimal investment path in terms of: i) its duration (as provided by the endogenous thresholds \tilde{t} , \underline{t} , \overline{t} and t^*), ii) for which goods is the productivity increased through investments in the respective capital stocks, and iii) for how many types of agents is effort exertion subsidized. Recall that if only one type of agent receives a taxation-break, then this is the type with the lowest bargaining power.

Case I:
$$\underline{Z_a} < \overline{Z_a} < \widetilde{Z_a}$$

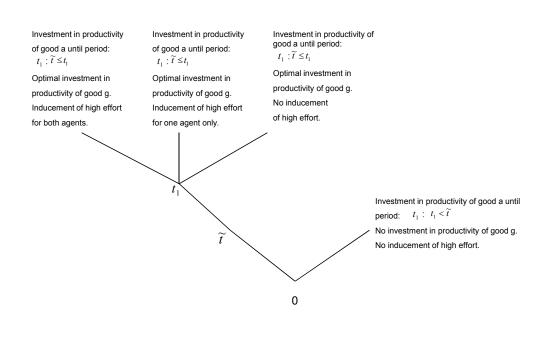


Figure 5: Case I

Case II: $\underline{Z_a} < \widetilde{Z}_a < \overline{Z_a}$

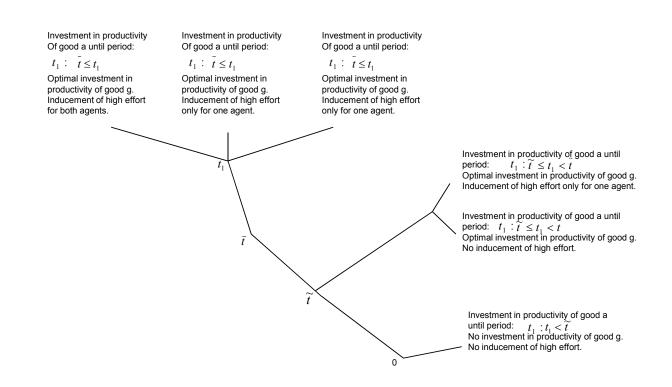


Figure 6: Case II

Case III:
$$\widetilde{Z}_a < \underline{Z}_a < \overline{Z}_a$$

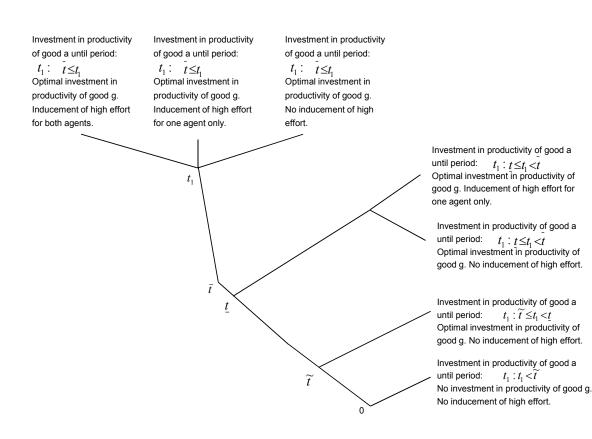


Figure 7: Case III