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Immigration, Public Pensions, and Heterogenous Voters

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Abstract

Depending on the design of the domestic pension system and the type of immigrants, voters will decide differently on immigration policy. In this paper, we investigate the voting outcome of three groups of heterogenous voters (skilled workers, unskilled workers, and retirees) under Beveridgian or Bismarckian pension systems which are either of the fixed contribution rate or the fixed replacement ratio type. We find that while the use of a Beveridge or Bismarck system does not change the results qualitatively, the fixed contribution rate vs. fixed replacement ratio distinction leads to substantial changes in the optimal choices of different groups.

Keywords: PAYG pension systems, Beveridge vs. Bismarck, immigration policy

JEL-Classification: H55, J61, D72

1 Introduction

The aging of societies in most industrialized countries has led to severe problems in financing public pension systems. One possible solution to these problems that has been proposed frequently in political and economic discussions is to allow immigrants into the countries. Immigration will increase the number of contributors to the pension system and thus alleviate the tax burden of domestic workers. Sinn (1997) estimates the positive net fiscal externality that an additional immigrant creates to the rest of the (German) society, i.e. the present value of lifetime contributions to the pension system, to be approximately 170,000 Euros. Among other things this effect rests on the fact that immigrant families have more children than German families. This means that immigrant children contribute more to the pension system than is needed to support their retired parents.

These results, however, are less clear-cut than they appear. In a simulation analysis, Razin/Sadka (2000) show, for example, that while the total welfare effect of *unskilled* immigration is positive, in most cases only the first generation's retirees gain by an increase in pension benefits. The following generations lose because of a downward pressure on wages due to an increasing number of unskilled workers[1]. Apart from this intergenerational effect, there are also differential intragenerational effects of immigration. The lower the skill level of workers, the more they are hurt by unskilled immigration because their marginal productivity of labor (and thus their wages) falls.

In this paper, we will formalize and extend the idea of having different effects of immigration on different groups in society. To do this, we consider the preferences with regard to the immigration of unskilled workers of three groups in society, namely skilled workers, unskilled workers, and retirees in a certain point of time. One can interpret this as a voting decision on immigration. The members of each group will choose the optimal levels of immigration by maximizing the expected income of their (remaining) lifetime, taking into account the effect of immigration on wages and pensions. This approach is similar to Scholten/Thum (1996) and Haupt/Peters

[1]The model assumes competitive labor markets. See also Kemnitz (2001) who introduces imperfect labor markets into this framework.

(1998) who use, however, models with just one skill group but three generations, namely young workers, middle-aged workers and retirees.

So far, the literature has not paid too much attention to the question of how to model the pension system. This is a problem because models on immigration policy and public pensions take the pension system as exogenously given. If one changes the underlying assumption about the pension system, the preferences with regard to immigration may change substantially. This effect has been emphasized by Haupt/Peters (1998) who show that in a pension system which fixes the contribution rate, immigration benefits the retirees because total contributions and thus pension benefits increase. In a fixed-replacement ratio regime, on the other hand, immigration is advantageous to the working generations because a constant total sum of pension benefits is financed by an increasing number of contributors. Hence, the contribution rate can be lowered. Most other papers assume just one of the two scenarios described before. Razin/Sadka (2000), for example, assume only fixed contribution rates while Scholten/Thum (1996) consider only a fixed-replacement ratio regime[2].

Because of the importance of the way the pension system is modelled, we will take up the distinction made by Haupt/Peters (1998). This means that we explicitly distinguish between pension systems which are either characterized by fixed contribution rates and endogenously determined replacement ratios or those with fixed replacement ratios and endogenous contribution rates. It should be noted that this distinction describes only polar cases. Most pension systems are not *purely* of one or the other type, but have relatively less variation in either one of the two parameters.

A second possible classification of pension systems is to distinguish between Beveridgian and Bismarckian systems (see, e.g., Bonoli (1997) or Cremer/Pestieau (1998)). Both pension systems are characterized by earnings-related contributions. In the Beveridgian system, however, we have a flat benefit in old age which is independent of previous earnings. Thus, the system *inter*generationally and *intra*generationally redistributive, i.e. workers with high incomes contribute relatively more to the flat benefit than workers with low income. In the *stylized* Bismarckian system, which we will employ in this paper, the individual pension benefit is related to individ-

[2]See Krieger (2002) for a survey of the recent literature on these topics.

ual earnings during working age[3]. While one would expect that this system is not redistributive, we will show this may nevertheless be the case.

The literature has not yet turned to the question whether the existence of Beveridgian and Bismarckian pension systems has different impacts on immigration policy. We will therefore investigate how the existence of either one of the two different pension systems changes to voting outcome.

Hence, in total we have four different scenarios which will be investigated. There is the Beveridgian pension system with either fixed contribution rate or fixed replacement ratio and there is the Bismarckian pension system with the same two possible features. The paper will therefore proceed as follows. In section 2, we introduce the basic model which consists of an economy with two factors of production, namely skilled and unskilled labor, and a specific pension system. In section 3, group-specific prefenrences (and hence the voting outcome) in a country with a Beveridgian pension system are investigated if the decision is to be made on the optimal level of unskilled immigration. In two subsections, we distinguish between the Beveridgian system with fixed contribution rates and the Beveridgian system with fixed replacement ratios. A similar distinction is also made in section 4 in which a Bismarckian pension system is investigated. Section 5 summarizes and concludes.

2 The theoretical framework

2.1 The economy

Consider a small open economy which has access to the international capital market, where the exogenous interest rate r_t prevails. The production function of the representative firm is a Cobb-Douglas aggregate of skilled and unskilled labor, H_t and L_t:

[3]Real-world Bismarckian pension systems are usually somewhat more complex. In Germany, for example, previous earnings are used to calculate "earnings points". In old age, the earnings points are related to recent net wages in order to determine the individual pension benefit. This method allows retirees to participate in the economy's productivity growth. We will, however, abstract from this feature by relating the pension benefit simply to previous earnings.

$$Y_t = H_t^{\alpha} \cdot L_t^{1-\alpha} \tag{1}$$

where $\dfrac{\partial F(\cdot)}{\partial H_t} > 0, \dfrac{\partial F(\cdot)}{\partial L_t} > 0, \dfrac{\partial^2 F(\cdot)}{\partial H_t^2} < 0, \dfrac{\partial^2 F(\cdot)}{\partial H_t^2} < 0$ and $\dfrac{\partial^2 F(\cdot)}{\partial H_t \cdot \partial L_t} > 0$.

Individual labor supply is given and normalized to one. Labor markets are competitive for both factors of production, so labor is paid its marginal product. The H_t skilled workers with a high level of productivity receive a wage w_t^H and the L_t unskilled workers with a low level of productivity receive w_t^L. Immigration of young workers (M_t^i, $i = H, L$) constitutes an increase in the size of the domestic groups. Hence, we have $\frac{\partial w_t^i}{\partial M^i} < 0$ and $\frac{\partial w_t^i}{\partial M_t^j} > 0 \; \forall \; i \neq j$. In the following, we will consider only the case of *unskilled* immigration which is constrained to be non-negative, i.e. the total number of unskilled workers is $L_t + M_t^L$ where $M_t^L \geq 0$ (and $M_t^H = 0$). Furthermore, we will assume immigration to take place only once, namely in period t.

The median wage is $w_m = w_t^L < \overline{w}$ as we assume $H_t < L_t$. The mean wage is

$$\overline{w}_t = \theta w_t^H + (1-\theta) w_t^L \tag{2}$$

with $\theta = \frac{H_t}{H_t + L_t}$ in the case without immigration and $\theta = \frac{H_t}{H_t + L_t + M_t^L}$ in the case with unskilled immigration. If we take (for the latter case) the derivative of $\overline{w}_t$ with respect to immigration M_t^L, we get

$$\frac{d\overline{w}_t}{dM_t^L} = \frac{H_t(w_t^L - w_t^H)}{H_t + L_t + M_t^L} < 0$$

since $w_t^L < w_t^H$ by assumption. Hence, unskilled immigration leads to falling average wages.

In each period t, there live two generations: workers and retirees. Population grows at rate n_t. This growth rate holds for all groups of the society, i.e. independent of people's skill level or origin. Hence, we have $H_t = (1+n_t)H_{t-1}$ and $L_t = (1+n_t)L_{t-1}$, respectively. Specifically, immigrants M_t^i, $i = H, L$, adopt the growth pattern of the domestic population immediately after entering the country. This assumption is frequently made in the literature, see e.g. Scholten/Thum (1996), Haupt/Peters (1998) or Razin/Sadka (1999, 2000).

Based on the post-migration production function

$$Y_t = H_t^{\alpha} \cdot \left(L_t + M_t^L\right)^{1-\alpha}, \tag{3}$$

let us note explicitly some relevant derivatives for further reference:

$$\frac{dw_t^H}{d\widetilde{M_t^L}} = \alpha\left(1-\alpha\right) H_{t-1}^{\alpha-1} \cdot \left(L_{t-1} + \widetilde{M_t^L}\right)^{-\alpha} > 0, \tag{4}$$

$$\frac{dw_t^L}{d\widetilde{M_t^L}} = -\alpha\left(1-\alpha\right) H_{t-1}^{\alpha} \cdot \left(L_{t-1} + \widetilde{M_t^L}\right)^{-\alpha-1} < 0 \tag{5}$$

where we define $\widetilde{M_t^i} := \frac{M_t^i}{1+n_t}$. One can easily see that $\lim_{\widetilde{M_t^L}\to\infty} \frac{dw_t^H}{d\widetilde{M_t^L}} = 0$.

2.2 The pension system

Two general types of pension systems will be considered in the following: Beveridgian and Bismarckian systems[4]. As we are talking about pay-as-you-go (PAYG) financed pension systems, it is assumed that the system's budget is to be balanced at all times, i.e. in each period t total contributions must equal total benefits.

In a Beveridgian system, in each period t the workers' contributions to the PAYG pension system are related to individual earnings w_t^i, $i = H, L$. The pension benefit paid to the retirees in the same period, however, depends on *average* earnings $\overline{w}_t$. Thus, the pension system's budget constraint before immigration can be written as

$$\tau_t \left(w_t^H H_t + w_t^L L_t\right) = q_t \overline{w}_t \left(H_{t-1} + L_{t-1}\right) \tag{6}$$

where q is the replacement ratio and τ the contribution rate to the pension system[5].

[4]The formal definition of Beveridgian and Bismarckian pension systems used in this paper follows Casamatta/Cremer/Pestieau (2000). We employ, however, a slight modification in the definition of the Bismarckian pension benefit, namely that the benefit is related to previous (instead of current) wages.

[5]We assume implicitly that the payroll tax rate of the Beveridgian system and the contribution rate of the Bismarckian system are equivalent. We will in both cases refer to it as contribution rate τ.

Let us define the *pension benefit per capita* to be $p_t = q_t \cdot \overline{w}_t$. After immigration has taken place we get

$$(1+n)\,\tau_t \left\{ w_t^H H_{t-1} + w_t^L \left(L_{t-1} + \widetilde{M_t^L} \right) \right\} = p_t \left(H_{t-1} + L_{t-1} \right).$$

Using the definition of $\overline{w}_t$ from equation (2), this simplifies to

$$(1+n)\,\tau_t \left\{ H_{t-1} + \left(L_{t-1} + \widetilde{M_t^L} \right) \right\} = q_t \left(H_{t-1} + L_{t-1} \right). \tag{7}$$

Notice that this equation does not depend on wages. This is a typical feature of Beveridgian pension systems in which only the relative size of generations determines the relation between contributions and benefits.

As in the Beveridgian system, contributions in a Bismarckian PAYG pension system are related to individual earnings in each period t. Pension benefits, however, now depend on the previous period's *individual* earnings, i.e. there are two types of retirees: those who were skilled workers before retirement and those who were unskilled before. Individual pension benefits are $q_t w_{t-1}^H$ and $q_t w_{t-1}^L$, respectively, i.e. we have a pension system with individual accounts. The pension system's budget constraint is

$$\tau_t \left\{ w_t^H H_t + w_t^L \left(L_t + M_t^L \right) \right\} = q_t \left(w_{t-1}^H H_{t-1} + w_{t-1}^L L_{t-1} \right). \tag{8}$$

after immigration. Notice that the same replacement ratio holds for all groups within one generation. This is a typical feature of a single mandatory (public) pension system which includes all groups in society. A (private) pension fund usually allows for differing contribution rates and sometimes also for differing replacement ratios for various groups of people (e.g. males/females).

The time structure of the model is the following. Individuals born in period $t-1$ retire in t and receive a pension benefit. Workers born in t contribute to the pension system. All contributions are transferred directly to the retirees. At the beginning of period t, i.e. before a new generation starts to work, all generations vote on the number of immigrants, taking into account the effect of this decision on their income over their remaining lifetime. Immigration according to the agreed policy takes place immediately after the vote. Production is then carried out with domestic workers and immigrants. Hence, immigration has an impact on all current variables, i.e. w_t^H, w_t^L, τ_t, q_t and p_t. Variables in $t-1$ and $t+1$ remain unchanged as we assumed immigration to take place only in period t[6]. Clearly, past variables cannot be changed anymore in a subsequent period. Neither do future variables change here because the relative size of groups of workers does not change from period t to $t+1$. This is because there is no future immigration[7] and because natives and immigrants have the same reproduction rate.

Furthermore, we assume that there is *no* impact on one of the current variables due to immigration if it is fixed by regulations of the pension system. Whether and how any one of these variables is involved in the individual optimization problem depends heavily on the type of pension system considered, i.e. whether we have a Beveridgian or Bismarckian system and whether we assume fixed contribution rates or a fixed replacement ratio.

3 Voting in a Beveridgian system

An important distinction in the organisation of pension systems is whether they tend to keep contribution rates fixed while endogenizing the replacement ratio (we call this the fixed-contribution rate or CR regime) or whether the systems predominantly fix the replacement ratio and let contribution rates adjust to demographic changes (fixed-replacement ratio or RR regime). In the following we discuss this distinction in a Beveridge system; the same distinction occurs in the Bismarckian system in the following section.

[6]It should be noted that in the case of a Beveridgian pension system this assumption can be interpreted as myopic behavior of the young generation. Future immigration decisions are made by future generations. So, today's workers are myopic in the sense of believing that their immigration decision does not have an impact on the next generation's immigration decision. They will not consider future decisions in their maximization problem. This does not hold in the Bismarckian system because future variables (e.g. the replacement ratio) are connected with today's variables (e.g. wages) which change through immigration. Hence, they have to be considered in today's workers decision problem.

[7]One gets the same result if the ratio of skilled and unskilled *immigrants* is the same as the ratio of skilled and unskilled *natives*.

3.1 Fixed-contribution rate regime

Under a fixed contribution rate regime, the replacement ratio q_t is endogenously determined while the contribution rate is fixed at level $\overline{\tau}$. This leads to the following equation which can be derived from the pension system's budget constraint (7):

$$q_t = \frac{\overline{\tau}\,(1+n_t)\left\{H_{t-1} + \left(L_{t-1} + \widetilde{M_t^L}\right)\right\}}{H_{t-1} + L_{t-1}} \tag{9}$$

where $\frac{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}{H_{t-1}+L_{t-1}}$ is analogous to Scholten/Thum's (1996) *immigration ratio* which is the ratio of labor supplies after and before immigration. For further reference we introduce the following notation. Let $m_t := \frac{\widetilde{M_t^L}}{H_{t-1}+L_{t-1}}$, then the immigration ratio equals $1+m_t$, which is the growth factor of labor supply due to unskilled immigration. Hence, we get $q_t = \overline{\tau}(1+n_t)(1+m_t)$ which means that the replacement ratio q_t grows due to reproduction of the domestic population and due to immigration.

Let us now consider the preferences of domestic groups with regard to unskilled immigration. Each member of the working generation maximizes his lifetime income[8] which is composed of current net wage income, i.e. $w_t^i(1-\overline{\tau})$, $i = H, L$, and the future pension benefit, p_{t+1}, given $\widetilde{M_t^L} \geq 0$:

$$\underset{\widetilde{M_t^L}}{max.}\;\; w_t^i(1-\overline{\tau}) + p_{t+1} \;\; s.t. \;\; \widetilde{M_t^L} \geq 0,\, i = H, L. \tag{10}$$

However, there will not be immigration in the future, hence, groups sizes as well as average wages do not change from t to $t+1$. Also, the replacement ratio remains unchanged. To see this, consider period $t+1$. From (9) we get

$$q_{t+1} = \frac{\overline{\tau}\,(H_{t+1} + L_{t+1})}{H_t + L_t + M_t^L}$$

where $H_{t+1} = (1+n_{t+1})H_t$ and $L_{t+1} = (1+n_{t+1})\left(L_t + M_t^L\right)$. So, we have

$$q_{t+1} = \overline{\tau}(1+n_{t+1})$$

[8]We assume the discount rate between two periods to be zero.

which is independent of immigration[9]. Notice the importance of the fact that immigrants behave just like domestic people. They are assumed to have the same fertility rate as natives and thus they have just as much offspring as to support themselves in old age. If fertility rates differ, n_{t+1} would possibly depend on current immigration[10]. Therefore, the optimization problem (10) reduces to maximizing $w_t^i(1-\overline{\tau})$ with respect to $\widetilde{M_t^L}$, assuming $\widetilde{M_t^L} \geq 0$. The (Kuhn-Tucker) first-order conditions are

$$\frac{d(w_t^i(1-\overline{\tau}))}{d\widetilde{M_t^L}} \leq 0, \quad \widetilde{M_t^L} \geq 0, \quad \frac{d(w_t^i(1-\overline{\tau}))}{d\widetilde{M_t^L}} \cdot \widetilde{M_t^L} = 0.$$

Under this scenario, the variation of w_t^i due to immigration is decisive as τ is fixed. From the underlying production function (3) we know that the marginal productivity of unskilled labor decreases, i.e. $\frac{\partial w_t^L}{\partial \widetilde{M_t^L}} < 0$. Hence, to meet the Kuhn-Tucker conditions, a zero-immigration policy is the preferred option for this group of workers ($\widetilde{M_t^L} = 0$). For skilled workers just the opposite is true as $\lim_{\widetilde{M_t^L} \to \infty} \frac{dw_t^H}{d\widetilde{M_t^L}} = 0$. They prefer the highest possible level of immigration, i.e. an infinite number of immigrants ($\widetilde{M_t^L} \to \infty$). The term "infinite" should not, however, be taken too literally. While $\widetilde{M_t^L} \to \infty$ is necessary to ensure that the Kuhn-Tucker conditions hold, in reality it is meant that this group prefers *unrestricted* or *unbounded* immigration. So, they simply reject any immigration laws and favor "open borders"[11].

Finally, the retirees have to be considered. They maximize their pension benefit p_t with respect to the level of immigration $\widetilde{M_t^L}$. Hence, we have

[9]This is also the reason why myopic voters will not consider the future pension benefit in their maximization problem. The benefit depends on the variables q_{t+1} and $\overline{w}_{t+1}$ which myopic workers believe they cannot influence. Hence, they will not include p_{t+1} in their optimization problem.

[10]If we assume a different growth rate for immigrants, we should expect a conflict between domestic and incoming persons. Let us assume, for example, that the immigrants have a lower population growth rate than the natives. Then, the pension funds per retiree raised in t+1 are lower than without immigration. Some of the pension benefits raised by the natives will be shifted to retired immigrants. This will have an impact on the voting outcome.

[11]Certainly, the number of immigrants will not be infinite because only a small fraction of foreign population is effectively mobile. This may be due to a strong attachment-to-home, falling income differentials or other reasons.

$$\underset{\widetilde{M_t^L}}{max.}\ q_t \cdot \overline{w}_t = \frac{\overline{\tau}(1+n_t)\left\{w_t^H H_{t-1} + w_t^L\left(L_{t-1} + \widetilde{M_t^L}\right)\right\}}{H_{t-1} + L_{t-1}} \quad s.t.\ \ \widetilde{M_t^L} \geq 0 \tag{11}$$

where we have employed (2) and (9). Analogously, the first-order (Kuhn-Tucker) conditions of this optimization problem are given by

$$\frac{d(q_t \cdot \overline{w}_t)}{d\widetilde{M_t^L}} \leq 0, \quad \widetilde{M_t^L} \geq 0, \quad \frac{d(q_t \cdot \overline{w}_t)}{d\widetilde{M_t^L}} \cdot \widetilde{M_t^L} = 0 \tag{12}$$

where

$$\frac{d(q_t \cdot \overline{w}_t)}{d\widetilde{M_t^L}} = w_t^L + \frac{dw_t^H}{d\widetilde{M_t^L}} H_{t-1} + \frac{dw_t^L}{d\widetilde{M_t^L}}\left(L_{t-1} + \widetilde{M_t^L}\right). \tag{13}$$

In equation (13)[12] the first term is the direct (positive) impact of additional contributors to the pension system's total funds. The second term is the indirect (positive) effect on the pension funds because wages of the skilled rise as they become relatively scarcer. The third term is the indirect decrease in contributions due to falling wages of the unskilled workers.

In the case of the Cobb-Douglas production function (1), we can easily see that the second and the third term cancel out (recall (4) and (5)), leaving us with $\frac{d(q_t \cdot \overline{w}_t)}{d\widetilde{M_t^L}} = (1-\alpha)\left(\frac{H_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{\alpha} \geq 0$. Only if $\widetilde{M_t^L}$ approaches infinity, the optimality condition (12) can be met ($\lim_{\widetilde{M_t^L}\to\infty} \frac{d(q_t \cdot \overline{w}_t)}{d\widetilde{M_t^L}} = 0$).

Hence, we find that the retirees vote in favor of unrestricted unskilled immigration. The reason for this is the positive effect on pensions due to an increasing number of contributors. We can interpret this result as an increase in the total sum of wages (given by (13)) due to the immigration of an additional unskilled worker. The positive effect on wages of the skilled workers and the negative effect on wages of

[12] We can also write this condition (in case it equals zero) in terms of elasticities. Then we get $\varepsilon_{LL} + \phi \cdot \varepsilon_{HL} = 1$ where $\varepsilon_{LL} = \frac{dw_t^L}{d\widetilde{M_t^L}} \frac{L_{t-1}+\widetilde{M_t^L}}{w_t^L}$ is the elasticity of the wage of *unskilled* workers with respect to *unskilled* immigration, $\varepsilon_{HL} = \frac{dw_t^H}{d\widetilde{M_t^L}} \frac{H_{t-1}}{w_t^H}$ is the elasticity of the wage of *skilled* workers with respect to *unskilled* immigration, and $\phi = \frac{w_t^H H_{t-1}}{w_t^L\left(L_{t-1}+\widetilde{M_t^L}\right)}$ is the ratio of total factor incomes of the groups.

the unskilled workers offset each other. Under this scenario, immigration is just a different form of fertility.

The political outcome depends on the assumed size of groups in society. We know that skilled workers and retirees are in favor of unrestricted unskilled immigration while unskilled workers are against it. It is reasonable to assume that no group in society has more than 50 percent of the votes, so unbounded immigration will be the voting outcome[13].

3.2 Fixed-replacement ratio regime

In the fixed-replacement ratio case just the opposite scenario of the previous case is assumed. The replacement ratio is fixed at a certain level while the contribution rate is endogenously determined. From the Beveridgian budget constraint (7) follows that

$$\tau_t = \frac{\overline{q}}{(1+n_t)} \frac{H_{t-1} + L_{t-1}}{H_{t-1} + L_{t-1} + \widetilde{M_t^L}} \tag{14}$$

if unskilled immigration takes place. With a fixed replacement ratio, an increasing number of contributors due to domestic reproduction and immigration leads to falling contribution rates. The effect on the preferences with regard to immigration is not unambiguous as we will see in the following. The reason is that unskilled workers face falling contribution rates as well as falling gross wages. A priori, it is not clear which effect dominates and thus whether this group's net income increases or falls.

This can be seen from the unskilled workers maximization problem. Again, each group of the working generation maximizes lifetime income in period t

$$w_t^i(1-\tau_t) + \overline{q} \cdot \overline{w}_{t+1}.$$

We assume that the replacement ratio is fixed at the same level in every period, i.e. $\overline{q} := \overline{q}_{(t)} = \overline{q}_{(t+1)}$. Hence, as in the CR regime the future pension benefit does

[13]There may be a situation in which the unskilled hold a majority of votes. In that case, zero immigration will be the preferred choice.

not play a role in the optimization problem of the workers. This is because the replacement ratio is fixed[14] and because the wage does not change from t to $t+1$.

There is, however, another difference compared to the CR regime: both the wage level and the contribution rate depend on $\widetilde{M_t^L}$. Given the restriction that $\widetilde{M_t^L} \geq 0$, the first of the unskilled worker's Kuhn-Tucker conditions is therefore given by

$$\frac{d\left(w_t^L(1-\tau_t)\right)}{d\widetilde{M_t^L}} = (1-\tau_t)\frac{dw_t^L}{d\widetilde{M_t^L}} + \frac{w_t^L \tau_t}{H_{t-1}+L_{t-1}+\widetilde{M_t^L}} \leq 0. \tag{15}$$

The first term in equation (15) is the negative effect on unskilled workers' wages because of $\frac{dw_t^L}{d\widetilde{M_t^L}} < 0$. The second term describes the fact that contributions per worker fall because, due to immigration, there is a higher number of workers. This term is positive, so we have two opposite effects. To show that we get an interior solution, i.e. $\frac{\partial\left(w_t^L(1-\tau_t)\right)}{\partial\widetilde{M_t^L}} = 0$ and $0 < \widetilde{M_t^L} < \infty$, we can rewrite (15) by using w_t^L from the Cobb-Douglas production function (3) and $\frac{dw_t^L}{d\widetilde{M_t^L}}$ from equation (5). We get the following expression (see the appendix):

$$\frac{d\left(w_t^L(1-\tau_t)\right)}{d\widetilde{M_t^L}} = 0 \Longleftrightarrow \overline{q} = \frac{\alpha\left(1+n_t\right)\left(1+m_t\right)^2}{1+\alpha\left(1+m_t\right)} \tag{16}$$

where we define $G^{BV}(m_t;\alpha,n) := \frac{\alpha(1+n_t)(1+m_t)^2}{1+\alpha(1+m_t)}$. In order to derive the right-hand side of (16) we substituted τ_t by $\overline{q}$ according to equation (14) and used the fact that $1+m_t = \frac{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}{H_{t-1}+L_{t-1}}$ (see the appendix). To be able to show the interior solution, we will turn to a graphical analysis (see Figure 1).

We find that additional immigration is the preferred policy choice of the unskilled workers if $\overline{q} > G^{BV}(m_t;\alpha,n)$ as it increases lifetime income ($\frac{d\left(w_t^L(1-\tau_t)\right)}{d\widetilde{M_t^L}} > 0$). If we fix the partial elasticity of output α and the fertility rate at a certain level, say $\overline{\alpha}$ and $\overline{n}$, we can draw $G^{BV}(m_t;\overline{\alpha},\overline{n})$ as a function of m_t which rises as we see from its derivative:

$$\frac{dG^{BV}(m_t;\overline{\alpha},\overline{n})}{dm_t} = \frac{\overline{\alpha}\left(1+\overline{n}\right)\left(1+m_t\right)\left[2+\overline{\alpha}\left(1+m_t\right)\right]}{\left[1+\overline{\alpha}\left(1+m_t\right)\right]^2} > 0$$

[14]Implicitly, a fixed replacement ratio implies that contribution rates may differ in both periods, i.e. $\tau_t \neq \tau_{t+1}$. This is, however, not a concern for the workers because it will only hurt the next generation.

The replacement ratio $\overline{q}$ is exogenously given. The $G^{BV}(m_t;\overline{\alpha},\overline{n})$ curve intersects $\overline{q}$ at the immigration rate m^*, which we call the critical level of m_t, from below[15]. At the critical level the positive effect of immigration on net wages turns into a negative one ($\frac{d\left(w_t^L(1-\tau_t)\right)}{d\widetilde{M_t^L}} < 0$), i.e. for $m_t < m^*$ the decrease in contribution rates overcompensates the decrease in gross wages whereas for $m_t > m^*$ the gross wage effect dominates. Hence, up to m^* there exists a preference for additional immigration. Any higher level of immigration than m^* will lead to a rejection of further immigration. A lower rate is preferable in this case. The critical level m^* must therefore be an interior solution. Only if $m_t = m^*$, there is no incentive to change the immigration rate. Hence, the Kuhn-Tucker condition holds with $\frac{\partial\left(w_t^L(1-\tau_t)\right)}{\partial\widetilde{M_t^L}} = 0$ and $0 < \widetilde{M_t^L} < \infty$.

INSERT FIGURE 1

To gain some more insights into the behavior of unskilled workers, we will look at some comparative statics, namely variations of $\overline{\alpha}$ and $\overline{n}$. Let us assume that $\overline{q} = G^{BV}$ holds initially, i.e. there is neither an advantage nor a disadvantage of immigration because net wages do not change. From (16) follows that $G^{BV}(m_t;\overline{\alpha},\overline{n})$ falls if either the reproduction rate $\overline{n}$ or the elasticity $\overline{\alpha}$ decreases. Let us consider such a variation. We find that the $G^{BV}(m_t;\overline{\alpha},\overline{n})$ curve shifts downwards and we get $\overline{q} > G^{BV}$ at m^*. So again, further immigration is wanted. The intuition for this result with regard to $\overline{n}$ becomes clear if one recalls the definition of the contribution rate. Rewriting equation (14), we have

$$\tau_t = \frac{\overline{q}}{(1+\overline{n})(1+m_t)}.$$

A decrease in the reproduction rate $\overline{n}$ increases the contribution rate because a constant total pension benefit is divided by fewer contributors. The higher contribution

[15]In principle, it is possible that $\overline{q} < G^{BV}(m;\alpha,n)$, i.e. no immigration incentive exists, for any level of immigration. For reasonable parameter values, however, we always get a point of intersection. If one evaluates (16) at $m_t = 0$, one finds that even for high values of $\alpha \in (0,1)$ or n plugged into $G^{BV}(m_t = 0;\alpha,n) = \frac{\alpha(1+n_t)}{1+\alpha}$ it gets difficult to attain reasonable levels of the replacement ratio as prevailing in many countries (such as 0.7 in Germany).

rate therefore leads *ceteris paribus* to a falling net wage $w_t^L(1-\tau_t)$. This negative effect on net wages can be offset by further immigration because this will increase the number of contributors and help to lower contributions per capita again. We can also consider an exogenous variation of the replacement ratio $\overline{q}$. At m^*, an increase in $\overline{q}$ shifts the $\overline{q}$ line above the G^{BV} curve, so $\overline{q} > G^{BV}$ and there exists a preference for immigration. This can be explained from (14) as well: a higher replacement ratio increases total pension benefits which have to be paid for by an unchanged number of contributors. Immigration then helps to relax the tax burden by increasing the number of contributors.

Let us briefly consider a numerical example. We assume zero-population growth which is not too unrealistic for today's industrialized countries. Also, we assume arbitrarily a partial elasticity of output (with respect to *skilled* labor) of 0.4. Hence, the critical value m^* is determined by

$$\overline{q} = \frac{0.4(1+m_t)^2}{1+0.4m_t}.$$

A replacement ratio of 70 percent leads to a critical immigration rate of approximately 0.45. This means that as long as the number of immigrants is less then 45 percent of domestic population for the given parameter values, there exists an incentive to vote in favor of more immigration because the effect of decreasing contribution rates more then offsets the decreasing gross wages. If immigration exceeds 45 percent, further immigration will be rejected. Notice that these numbers refer to generations, i.e. the preferred level of immigration is 45 percent in approximately *30 years.*

Turning to the skilled workers we receive an unambiguous result. Maximization of $w_t^H(1-\tau_t)$ leads to (Kuhn-Tucker) first-order conditions which have been derived in the same way as in the case of unskilled workers, except for using w_t^H instead of w_t^L. The main difference to (15) is that now the first term is positive since $\frac{dw_t^H}{d\widetilde{M}_t^L} > 0$. Skilled workers gain twice: first by an increase in their wages as they become a relatively scarcer factor and second by lower contribution rates. Net wages therefore increase unambiguously and skilled workers are in favor of unskilled immigration. The Kuhn-Tucker conditions therefore hold with

$\frac{d\left(w_t^H(1-\tau_t)\right)}{d\widetilde{M_t^L}} = (1-\tau_t)\frac{dw_t^H}{d\widetilde{M_t^L}} + \frac{w_t^H \tau_t}{H_{t-1}+L_{t-1}+\widetilde{M_t^L}} = 0$ and $\widetilde{M_t^L} \to \infty$ which is the only permissible solution as $\frac{d\left(w_t^H(1-\tau_t)\right)}{d\widetilde{M_t^L}}$ is positive for any other value of $\widetilde{M_t^L}$.

Finally, we have to consider the retirees who maximize their pension benefits, i.e. $p_t = \overline{q} \cdot \overline{w}_t$, with respect to immigration. Using the definition of the average wage (2), we get

$$\frac{dp_t}{d\widetilde{M_t^L}} = \overline{q} \cdot \frac{H_t \left(w_t^L - w_t^H\right)}{\left(H_t + L_t + M_t^L\right)^2} < 0 \tag{17}$$

because $w_t^L < w_t^H$. Although the replacement ratio is fixed, the pension benefit is not because it depends on the average wage. Due to unskilled immigration, the average wage falls. The Kuhn-Tucker conditions turn out to be $\frac{dp_t}{d\widetilde{M_t^L}} < 0$, $\widetilde{M_t^L} = 0$, and $\frac{dp_t}{d\widetilde{M_t^L}} \cdot \widetilde{M_t^L} = 0$. Retirees will therefore vote against immigration.

Falling average wages go along with decreasing total pension benefits. Both skilled and unskilled workers gain at the expense of the retirees. In a model *without* different skill groups, the (single) young generation will clearly be in favor of immigration. In our model, however, the fact that there exist skilled and unskilled workers whose wages react differently on immigration leads to an interior solution, i.e. the voting outcome is a restricted, but positive level of immigration. The decisive group in this scenario is the group of unskilled workers which has to weigh the two opposite effects describe above. This group therefore chooses a number of immigrants which corresponds to the the critical level m^* which we derived graphically. By the median voter theorem this level must be the voting outcome.

We summarize the results of section 3 in the Table 1.

Beveridge	CR	RR
Unskilled workers	zero	restricted ($0 < \widetilde{M_t^L}^* < \infty$)
Skilled workers	unrestricted	unrestricted
Retirees	unrestricted	zero
Voting outcome	unrestricted	restricted ($0 < \widetilde{M_t^L}^* < \infty$)

Table 1: Preferred level of immigration and voting outcome in the Beveridgian system

While the group of skilled workers gains in any case from unskilled immigration, comparing the CR and the RR regime, we find that a change from one system to the other will turn the incentive to vote in favor of immigration upside down. In the CR regime the retirees gain from increasing pension benefits while the unskilled workers face falling wages. Just the oppostite holds for the RR regime: the retirees now lose from falling average wages which are used to calculate their pension benefits. The unskilled workers still face falling wages but gain at the same time from falling contribution rates. From their point of few, restricted immigration (up to an optimal level) is favorable.

4 Voting in a Bismarckian system

4.1 Fixed-contribution rate regime

From the budget constraint of the Bismarckian pension system (8), we derive the following equation for the replacement ratio under a fixed-contribution rate regime:

$$q_t = \frac{\overline{\tau}(1+n_t)\left(w_t^H H_{t-1} + w_t^L\left(L_{t-1} + \widetilde{M_t^L}\right)\right)}{w_{t-1}^H H_{t-1} + w_{t-1}^L L_{t-1}} \tag{18}$$

Again, workers maximize today's net income, but they also have to take into account that their expected pension benefits in the second period is related to today's gross wages w_t^i, which depend on immigration today. Under the restriction that $\widetilde{M_t^L}$ is non-negative, the optimization problem is

$$\underset{\widetilde{M_t^L}}{max.}\;\; w_t^i(1-\overline{\tau}) + q_{t+1}\cdot w_t^i \;\; s.t.\;\; \widetilde{M_t^L} \geq 0,\, i = H, L. \tag{19}$$

It is important to investigate how the next period's replacement ratio q_{t+1} behaves. According to (18), we get

$$q_{t+1} = \frac{\overline{\tau}\left(w_{t+1}^H H_{t+1} + w_{t+1}^L L_{t+1}\right)}{w_t^H H_t + w_t^L\left(L_t + M_t^L\right)} \tag{20}$$

where $H_{t+1} = (1+n_{t+1})H_t$ and $L_{t+1} = (1+n_{t+1})\left(L_t + M_t^L\right)$. Recall that we assumed no future immigration and that all groups of workers (including immigrants) have the same fertility rate. This means that the relative sizes of groups do not change from period t to period $t+1$. Hence, the marginal productivities of skilled and unskilled labor remain unchanged as well, so we get $w_t^i = w_{t+1}^i$. In (20) the bracketed terms can therefore be cancelled, leaving us with $q_{t+1} = (1+n_{t+1})\overline{\tau}$. The unskilled worker's lifetime income therefore turns out to be

$$w_t^L(1-\overline{\tau}) + (1+n_{t+1})\overline{\tau}w_t^L = w_t^L(1+\overline{\tau}n_{t+1}), \tag{21}$$

i.e. it consists of labor income and the contribution to the pension system compounded by the *biological* interest rate n_{t+1}.

Maximizing (21) with respect to unskilled immigration shows that zero immigration is the unskilled worker's preferred policy option because the Kuhn-Tucker conditions are met with $\frac{dw_t^L}{d\widetilde{M_t^L}} < 0$ and $\widetilde{M_t^L} = 0$. The skilled worker has a similar optimization problem. He maximizes $w_t^H(1+\overline{\tau}n_{t+1})$ with $\frac{dw_t^H}{d\widetilde{M_t^L}} \geq 0$ where $\lim_{\widetilde{M_t^L}\to\infty} \frac{dw_t^H}{d\widetilde{M_t^L}} = 0$, so the only solution to the optimzation problem is to have maximum immigration ($\widetilde{M_t^L} \to \infty$).

The retirees simply maximize their expected pension benefit after immigration, which is $p_t^i = q_t \cdot w_{t-1}^i$, $i = H, L$. It is necessary to use the i-index on the pension variable because the Bismarckian system is characterized by individual accounts. The pension benefit of formerly skilled retirees is related to the previous period's wages for the skilled. For formerly unskilled workers the analogous argument applies. Clearly, we have to take into account that both groups may vote differently on immigration policy.

Let us now turn to the first of the Kuhn-Tucker conditions for each of the two groups. We get

$$\frac{\overline{\tau}(1+n)}{w_{t-1}^H H_{t-1} + w_{t-1}^L L_{t-1}} \left(w_t^L + \frac{dw_t^H}{d\widetilde{M_t^L}} H_{t-1} + \frac{dw_t^L}{d\widetilde{M_t^L}} \left(L_{t-1} + \widetilde{M_t^L} \right) \right) w_{t-1}^i \leq 0 \tag{22}$$

for $i = H, L$. Dividing by the first and the last term, which are both positive, leaves us with the bracketed expression which is just the same as equation (13), the first-order condition of the retirees in the Beveridgian system. The same interpretation as in (13) applies. As the second and the third term cancel out under the Cobb-Douglas scenario given by (3), we get $\frac{d(q_t \cdot w_{t-1}^i)}{d\widetilde{M_t^L}} = (1-\alpha)\left(\frac{H_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^\alpha \geq 0$ which approaches zero if $\widetilde{M_t^L} \to \infty$. The Kuhn-Tucker conditions are met with $\frac{d(q_t \cdot w_{t-1}^i)}{d\widetilde{M_t^L}} = 0$, $i = H, L$, and $\widetilde{M_t^L} \to \infty$.

Notice that all group-specific terms have cancelled out in (22). Hence, both skilled and unskilled retirees will vote in the same way, i.e. they are in favor of unrestricted immigration. So, we can join the two groups of retirees to just one group as in the Beveridgian pension system. The reason for this surprising result is that pension benefits are calculated with respect to wages in the pre-immigration period $t-1$. Immigration in t has an impact only on the replacement ratio q_t which improves. Because q_t is the same for all retirees, the optimal $\widetilde{M_t^L}^*$ guarantees that both $p_t^H = q_t w_{t-1}^H$ and $p_t^L = q_t w_{t-1}^L$ are maximized. Hence, the pension system is Bismarckian only in the sense that the basis to calculate pensions, which is the wage from the previous period, differs between workers. We can therefore conclude that there is no major difference in the outcome between Beveridgian and Bismarckian pension systems under the fixed-contribution rate regime. Theoretically, there is no need to explicitly distinguish between the two systems.

There is a further implication of the model that should be noted. We would expect that in our stylized Bismarckian system with individual accounts no *intra*generational redistribution takes place. However, this may not be true because of a transmission effect via the joint replacement ratio q_t. Let us imagine that there exist two distinct pension systems for skilled and unskilled workers with different replacement ratios, say q_t^H and q_t^L. If unskilled immigration takes place in t, the wages of the skilled workers increase. With fixed contribution rates, total contributions of the skilled and therefore the fictitious replacement ratio q_t^H increase. Formerly skilled retirees gain from this effect because $p_t^H = q_t^H w_{t-1}^H$ increases. At the same time the wages of the unskilled workers decrease. In terms of total contributions of the unskilled, this negative effect is partially compensated by a higher number of contributors due to immigration. We can expect that in general the fictitious replacement ratio q_t^L of the unskilled workers differs from q_t^H (this is not the case in the Cobb-Douglas scenario).

Assuming now that both groups are forced into a joint pension system, although replacement ratios differ initially, shows that intragenerational redistribution between formerly skilled retirees and formerly unskilled retirees takes place even in the stylized Bismarckian system. Consider $q_t^H > q_t > q_t^L$, then redistribution takes place from skilled to unskilled workers. The pension benefit of the skilled decreases from p_t^H to $p_t^{H\prime} = q_t w_{t-1}^H$, while it increases for the unskilled.

4.2 Fixed-replacement ratio regime

Fixing the replacement ratio turns the system's budget constraint to

$$\tau_t = \frac{\overline{q}\left(w_{t-1}^H H_{t-1} + w_{t-1}^L L_{t-1}\right)}{(1+n_t)\left(w_t^H H_{t-1} + w_t^L\left(L_{t-1} + \widetilde{M_t^L}\right)\right)}. \tag{23}$$

The fixed replacement ratio goes along with different contribution rates for different periods. We have $\overline{q}_t = \tau_t(1+n_t)$ and $\overline{q}_{t+1} = \tau_{t+1}(1+n_{t+1})$ with $\overline{q} = \overline{q}_t = \overline{q}_{t+1}$ and $\tau_t \neq \tau_{t+1}$. The differing contribution rates are of no concern for today's generation, however. Workers in period t maximize their lifetime income

$$w_t^i(1-\tau_t) + \overline{q}_{(t+1)} w_t^i = w_t^i(1+\overline{q}-\tau_t),\ \ i = H, L \tag{24}$$

with respect to unskilled immigration which is again restricted to be non-negative ($\widetilde{M_t^L} \geq 0$). From (24) we first derive the optimal choice of an unskilled worker by deriving the first of the relevant Kuhn–Tucker conditions:

$$\frac{d\left(w_t^L(1+\overline{q}-\tau_t)\right)}{d\widetilde{M_t^L}} = (1+\overline{q}-\tau_t)\frac{dw_t^L}{d\widetilde{M_t^L}} + w_t^L\frac{d(1+\overline{q}-\tau_t)}{d\widetilde{M_t^L}} \leq 0. \tag{25}$$

We can determine three effects from this condition. The first term combines two negative effects from falling wages due to immigration. On the one hand, today's earnings decrease. On the other hand, future pension benefits fall because the basis to calculate them, i.e. today's gross wages, decreases while the replacement ratio remains unchanged. The second term describes the only positive effect from immigration which is the lower contribution rate. To show that we get an interior solution, i.e. $\frac{d\left(w_t^L(1+\overline{q}-\tau_t)\right)}{d\widetilde{M_t^L}} = 0$ and $0 < \widetilde{M_t^L} < \infty$, we rewrite equation (25) as

$$(1+\overline{q}-\tau_t)\frac{dw_t^L}{d\widetilde{M_t^L}} \leq -\tau_t w_t^L \frac{\frac{dw_t^H}{d\widetilde{M^L}}H_{t-1}+\frac{dw_t^L}{d\widetilde{M^L}}\left(L_{t-1}+\widetilde{M_t^L}\right)+w_t^L}{w_t^H H_{t-1}+w_t^L\left(L_{t-1}+\widetilde{M_t^L}\right)} \tag{26}$$

where τ_t is given by (23). Plugging w_t^L, w_t^H, $\frac{dw_t^L}{d\widetilde{M_t^L}}$, and $\frac{dw_t^H}{d\widetilde{M^L}}$ from production function (3) into (26), we get after some calculations the following condition (see the appendix):

$$\frac{d\left(w_t^L(1+\overline{q}-\tau_t)\right)}{d\widetilde{M_t^L}}=0 \Longleftrightarrow \overline{q}=\frac{\alpha(1+n_t)}{\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha}-\alpha(1+n_t)}. \tag{27}$$

As in section 3.2, we get a condition which tells us under which circumstances lifetime income increases due to a marginal increase in unskilled immigration. Again, we will employ a graphical analysis. We define the ratio on the right-hand side as a function $G^{BS}(\widehat{m}_t;\overline{\alpha},\overline{n}) := \frac{\overline{\alpha}(1+\overline{n})}{(1+\widehat{m}_t)^{-(1-\overline{\alpha})}-\overline{\alpha}(1+\overline{n})}$ where $1+\widehat{m}_t=\frac{L_{t-1}+\widetilde{M_t^L}}{L_{t-1}}$. We may call the latter term the *unskilled immigration ratio* (in contrast to the definition of m_t which is the total immigration ratio).

Equation (27) is much more complex than the corresponding condition (16) from the Beveridgian scenario. This can also be seen from Figure 3[16]. For a given α the function $G^{BS}(\widehat{m}_t;\overline{\alpha},\overline{n})$ has two branches, one of them lying in the lower right quadrant of the coordinate system. The explanation for this phenomena can be found in (27): if immigration gets very large, eventually the immigration ratio in the denominator becomes smaller than $\overline{\alpha}(1+\overline{n})$. Then $G^{BS}(\widehat{m}_t;\overline{\alpha},\overline{n})$ turns negative. We do not consider this branch as it has no reasonable economic meaning. This leaves us with the branch of the function that lies in the positive quadrant. For low values of $\widehat{m}_t$ (which can still be some one-hundred percent), $G^{BS}(\widehat{m}_t;\overline{\alpha},\overline{n})$ increases with unskilled immigration. As long as immigration is below a critical value $\widehat{m}^*$ (which corresponds to $\widetilde{M_t^L}^{*\prime}$), the positive effect from decreasing contribution rates due to immigration overcompensates the two negative effects related to falling gross wages. If, however, $\widehat{m}_t > \widehat{m}^*$ the negative effects dominate and zero immigration is the voting outcome. Hence, we get basically the same result as in the Beveridgian regime:

[16]In order to plot this graphic we made w.l.o.g. a simplifying assumption, namely, we set $L_{t-1} = 1$. Hence, $\widetilde{M_t^L}$ is just some small value.

for sufficiently low immigration rates, unskilled workers are in favor of additional immigration. For too high rates, they vote against further immigration. We therefore get an interior solution with some optimal positive level of immigration, so the Kuhn-Tucker conditions are met. Unfortunately, we are not able to compare the results from both section as our measures of immigration are not the same (m versus $\widehat{m}$).

However, these results depend critically on the chosen parameter values. From Figure 3, we can conclude that too high values of the output elasticity α lead to a situation in which no point of intersection exists. Then, $\overline{q} < G^{BS}(\widehat{m}_t; \overline{\alpha}, \overline{n})$ for any level of immigration. Hence, the Kuhn-Tucker conditions will be met only with $\frac{d\left(w_t^L(1+\overline{q}-\tau_t)\right)}{d\widetilde{M_t^L}} < 0$ and $\widetilde{M_t^L} = 0$. The optimal choice of unskilled workers is therefore zero immigration. In Figure 3, this problem occurs, e.g., for $\alpha = 0.6$. Except for this case, the comparative statics show the same results as in the Beveridgian scenario: a lower birth rate, a lower elasticity α or a higher replacement ratio make *c.p.* the pension system costlier to domestic population. The curves shift accordingly. Hence, there is an incentive to vote in favor of additional immigration.

Turning to the skilled workers, we have a similar first-order condition as (25) except for the fact that w_t^L is to be substituted by w_t^H. Using the same approach as before, we end up with the following (first) Kuhn-Tucker condition for the optimal choice of skilled workers:

$$\frac{d\left(w_t^H(1+\overline{q}-\tau_t)\right)}{d\widetilde{M_t^L}} = (1+\overline{q}-\tau_t)\frac{dw_t^H}{d\widetilde{M_t^L}} + w_t^H\frac{d(1+\overline{q}-\tau_t)}{d\widetilde{M_t^L}} \leq 0. \tag{28}$$

Both terms are non-negative. It is obvious that the first term becomes zero if immigration approaches infinity. Employing the Cobb-Douglas production function (3), we find that this holds for the second term, too. We get

$$\lim_{\widetilde{M_t^L}\to\infty} w_t^H\frac{d(1+\overline{q}-\tau_t)}{d\widetilde{M_t^L}} = \frac{\alpha(1-\alpha)\overline{q}}{1+n_t}\frac{H_{t-1}^{\alpha-1}L_{t-1}^{1-\alpha}}{L_{t-1}+\widetilde{M_t^L}} = 0.$$

Hence, we can conclude that the Kuhn-Tucker conditions holds only for $\widetilde{M_t^L}^* \to \infty$, i.e. skilled workers are in favor of unrestricted unskilled immigration. The reasoning is analogous to the case of unskilled workers, however, there are three *positive* effects here. The first two effects stem from the fact that skilled labor becomes scarcer and therefore wages increase. Hence, today's earnings and future pension benefits increase. The third effect is again lower contribution rates.

The retirees maximize their pension benefit which is $p_t^L = \overline{q} \cdot w_{t-1}^L$ or $p_t^H = \overline{q} \cdot w_{t-1}^H$, respectively. Since the replacement ratio is fixed at $\overline{q}$ and the previous period's wages are not affected by today's immigration, retirees from both groups remain indifferent. They neither gain nor lose from unskilled immigration. We can therefore assume that they either do not participate in the vote or that they toss a coin, i.e. on average one half of them will vote in favor of and the other half against immigration. Hence, the voting outcome depends only on the relative sizes of the groups. As we assumed that there are more unskilled than skilled workers, the unskilled workers will be the majority and vote against immigration.

The results from section 4 are summerized in the following table.

Bismarck	CR	RR
Unskilled workers	zero	restricted ($0 \leq \widetilde{M_t^L}^{*\prime} < \infty$)
Skilled workers	unrestricted	unrestricted
Retirees (formerly skilled)	unrestricted	indifferent
Retirees (formerly unskilled)	unrestricted	indifferent
Voting outcome	unrestricted	restricted ($0 \leq \widetilde{M_t^L}^{*\prime} < \infty$)

Table 2: Preferred level of immigration and voting outcome in the Bismarckian system

Comparing the results from this section with those from section 3.2, we find that skilled workers are clearly in favor of unskilled immigration because they will gain in any case, i.e. due to both higher wages and lower contribution rates. In the Bismarckian pension system they additionally gain from increasing pension benefits. The retirees behave differently depending on the assumed pension system: in the Beveridgian system they vote against immigration because the average wage which is the basis to calculate pension benefits falls. In the Bismarckian system the basis of calculation is the previous period's wage which remains unaffected by immigration. Therefore, Bismarckian retirees are indifferent with regard to unskilled immigration.

Though the argument is slightly different for the Beveridgian and the Bismarckian pension system, we find that the unskilled workers are decisive for the voting outcome. In the Beveridgian system, the median voters comes from the unskilled group while in the Bismarckian system they are the majority because the retirees do not participate in the election. So far, we can only make a qualitative argument with regard to the behavior of unskilled workers under both pension systems. In principle, under both scenarios there are positive and negative effects of unskilled immigration on lifetime income. Unskilled workers gain from falling contribution rates and lose from falling gross wages. Assuming that all other variables are the same under both regimes, we would expect that the additional negative wage effect on future pension benefits will make unskilled workers vote in favor of even less immigration under the Bismarckian system compared to the Beveridgian system. Partial evidence for this conclusion can be derived from the fact that under the Bismarckian system it is possible to have a zero immigration outcome while this is very unlikely to occur in the Beveridgian system if one assumes reasonable parameter values.

5 Conclusions

In this paper we investigated the impact that different pension systems have on the outcome of a voting decision on immigration policy. We assumed a median-voter model with three groups of voters, namely skilled workers, unskilled workers and retirees. Pension systems were either Beveridgian or Bismarckian with either fixed contribution rates or fixed replacement ratios.

The results that we find are the following. Under a fixed contribution rate regime, the increasing number of contributors due to immigration increases total revenues which can be used to pay higher pension benefits to retirees. The working generations either gain or lose from immigration depending on whether they become the scarcer or the more extensive factor of production. As we assumed unskilled immigration, unskilled workers are worse off from immigration while skilled workers are better off. These results hold regardless whether we assume a Beveridgian or a Bismarckian pension system. Hence, we find that in both cases there is a majority of voters (skilled workers and retirees) who vote in favor of immigration while only the unskilled workers vote against it. The voting outcome is to have unrestricted unskilled immigration.

Under the fixed-replacement ratio regime things become less obvious. Given the total pension benefit in this case, immigration leads to falling contribution rates. Skilled workers will clearly gain from unskilled immigration because of lower contribution rates and higher wages as they become the scarcer factor of production. A fixed replacement ratio does not in any case guarantee unchanged pension benefits. Under the Beveridgian scenario benefits are calculated with respect to average wages which fall due to immigration. Here, retirees are against immigration while Bismarckian retirees remain indifferent.

The decisive voter is the unskilled worker who faces opposing effects from unskilled immigration. If immigration is rather low, the positive effect from falling contribution rates dominates the negative gross wage effect. If immigration is high, the opposite holds. The level of immigration will therefore be raised until both effects offset each other, i.e. there exists an optimal positive level of immigration. The question in which pension system, the Beveridgian or the Bismarckian, the optimal level is higher can not yet be answered and will be a topic of further research.

We can conclude that it makes a substantial difference whether one considers a pension system with fixed contribution rates or with a fixed replacement ratio. Incentives to vote in favor or against immigration change for different groups under one or the other regime. At the same time, it does not make too much of a difference whether the system is Beveridgian or Bismarckian. The general voting outcome and the incentives of the groups remain basically unchanged. This is an important implication for research on this topic: while we can still work with the (mostly used) Beveridgian pension system and do not necessarily have to take into consideration the Bismarckian case, we have be extremly careful whether we employ a fixed contribution rate or a fixed replacement ratio regime.

There is a need for further research on the topics discussed in this paper. First, the impact of differing fertility rates of natives and immigrants on the voting outcome needs to be investigated. So far it was assumed that all groups have the same fertility rate which does not necessarily fit the real world data. Furthermore, the overall "quality" of immigration, i.e. the average skill level, should be endogenized, assuming that immigrants of different skill levels are allowed into the country at the same time.

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7 Appendix

Derivation of equation (16):

Plug w_t^L, $\frac{dw_t^L}{d\widetilde{M_t^L}}$ and τ_t according to (14) into (15):

$$\begin{aligned}
&\left(1-\frac{\overline{q}}{(1+n_t)}\frac{H_{t-1}+L_{t-1}}{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}\right)\alpha\,(1-\alpha)\,H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha-1} \\
= \quad & \frac{(1-\alpha)\,H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha}}{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}\frac{\overline{q}}{(1+n_t)}\frac{H_{t-1}+L_{t-1}}{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}.
\end{aligned}$$

Make use of $1+m_t=\frac{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}{H_{t-1}+L_{t-1}}$ and cancel terms. Then,

$$\alpha\left(1-\frac{\overline{q}}{(1+n_t)\,(1+m_t)}\right)\left(L_{t-1}+\widetilde{M_t^L}\right)^{-1}=\frac{1}{H_{t-1}+L_{t-1}+\widetilde{M_t^L}}\frac{\overline{q}}{(1+n_t)\,(1+m_t)}$$

which is

$$\alpha\left((1+n_t)\,(1+m_t)-\overline{q}\right)=\frac{\overline{q}}{(1+m_t)}$$

or

$$\overline{q}=\frac{\alpha\,(1+n_t)\,(1+m_t)^2}{1+\alpha\,(1+m_t)}. \qquad \square$$

Derivation of equation (27):

Plug w_t^H, w_{t-1}^H, w_t^L and w_{t-1}^L into (23):

$$\begin{aligned}
\tau_t &= \frac{\overline{q}\left(w_{t-1}^H H_{t-1}+w_{t-1}^L L_{t-1}\right)}{(1+n_t)\left(w_t^H H_{t-1}+w_t^L\left(L_{t-1}+\widetilde{M_t^L}\right)\right)} \\
&= \frac{\overline{q}H_{t-1}^{\alpha}L_{t-1}^{1-\alpha}}{(1+n_t)H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{1-\alpha}} \\
&= \frac{\overline{q}}{(1+n_t)}\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha}.
\end{aligned}$$

Together with w_t^H, w_t^L, $\frac{dw_t^H}{d\widetilde{M_t^L}}$ and $\frac{dw_t^L}{d\widetilde{M_t^L}}$, this can be plugged into (26). Hence,

$$\frac{d\left(w_t^L(1+\overline{q}-\tau_t)\right)}{d\widetilde{M_t^L}} = 0$$

$$\begin{aligned}
\iff \quad & -\left[1+\overline{q}-\frac{\overline{q}}{(1+n_t)}\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha}\right]\alpha\,(1-\alpha)\,H_{t-1}^{\alpha}\cdot\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha-1} \\
= \quad & -\frac{\overline{q}}{(1+n_t)}\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha}(1-\alpha)\,H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha} \\
& \cdot\frac{\left\{\begin{array}{c}\alpha(1-\alpha)H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha}-\alpha(1-\alpha)H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha}\\ +(1-\alpha)H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{-\alpha}\end{array}\right\}}{\alpha H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{1-\alpha}+(1-\alpha)\,H_{t-1}^{\alpha}\left(L_{t-1}+\widetilde{M_t^L}\right)^{1-\alpha}}
\end{aligned}$$

Cancelling terms leaves us with

$$\alpha\,(1+\overline{q})-\alpha\frac{\overline{q}}{(1+n_t)}\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha} = (1-\alpha)\frac{\overline{q}}{(1+n_t)}\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha}$$

$$\begin{aligned}
\iff \quad & \alpha\,(1+\overline{q}) = \frac{\overline{q}}{(1+n_t)}\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha} \\
\iff \quad & \overline{q} = \frac{\alpha(1+n_t)}{\left(\frac{L_{t-1}}{L_{t-1}+\widetilde{M_t^L}}\right)^{1-\alpha}-\alpha(1+n_t)}. \qquad \square
\end{aligned}$$

Figure 1: The unskilled workers' decision (Beveridge RR regime)

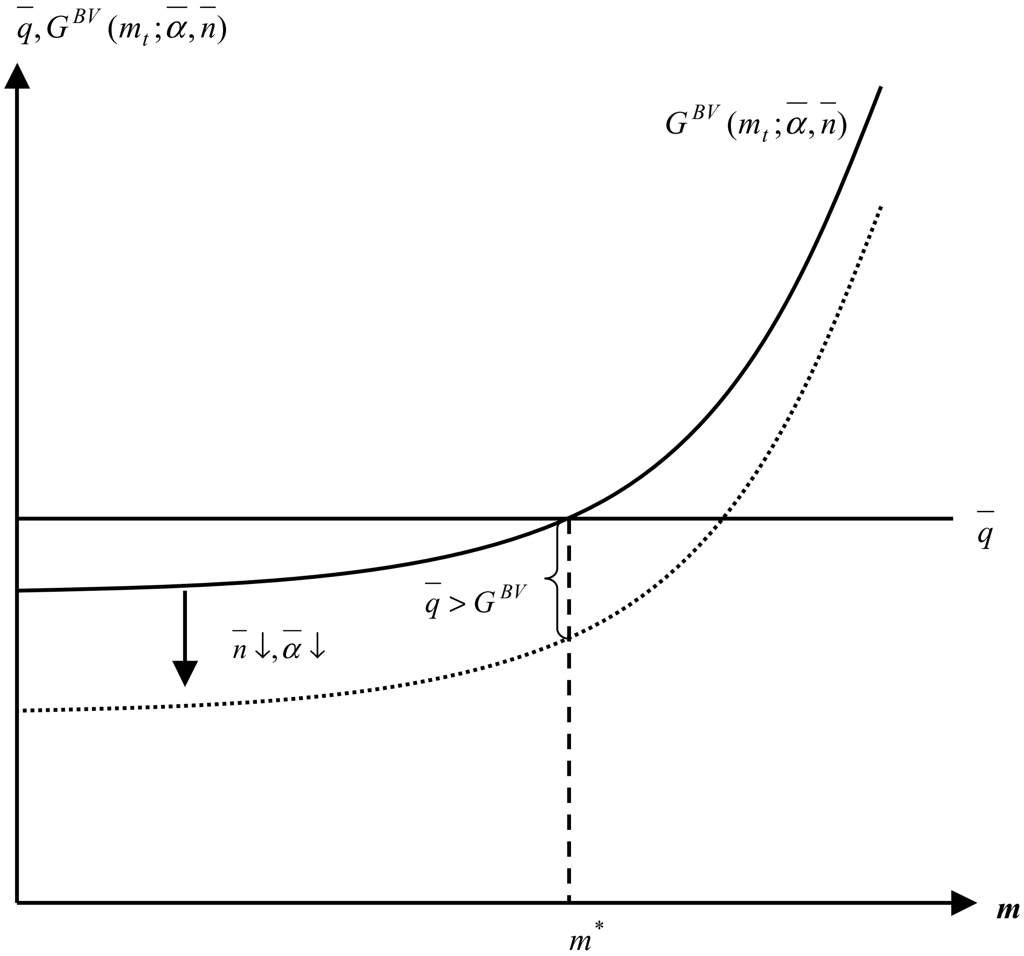
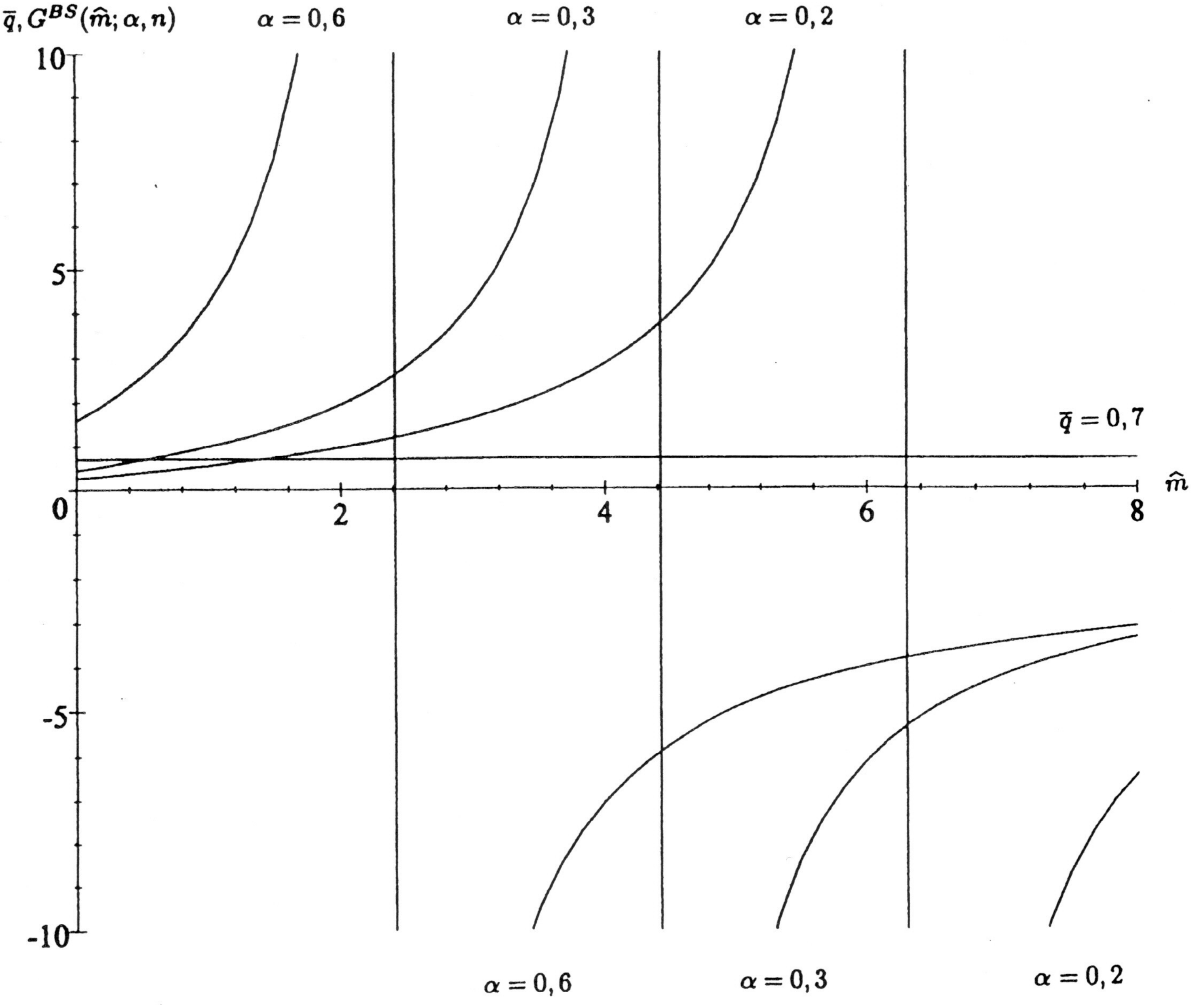


Figure 2: The unskilled workers' decision (Bismarck RR regime)



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